Complex numbers 29.4.2020

[41 marks]

Solve $z^2 = 4\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}$, giving your answers in the form

1a. $r\mathrm{e}^{\mathrm{i}\theta}$ where r, $\theta \in \mathbb{R}$, $r > 0$. *[3 marks]*

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1b. $a + \mathrm{i}b$ where a, $b \in \mathbb{R}$. *[2 marks]*

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Consider the complex number $z = \frac{2+7\text{i}}{6+2\text{i}}$.

2a. Express z in the form $a + \text{i}b$, where $a,\ b \in \mathbb{Q}$. *[2 marks]*

2b. Find the exact value of the modulus of z. *[2 marks]*

2c. Find the argument of z, giving your answer to 4 decimal places. *[2 marks]*

3a. Find three distinct roots of the equation $8z^3 + 27 = 0,\ z \in \mathbb{C}$ giving your answers in modulus-argument form. *[6 marks]*

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3b. The roots are represented by the vertices of a triangle in an Argand diagram. *[3 marks]*

Show that the area of the triangle is $\frac{27\sqrt{3}}{16}$.

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4. Consider the complex numbers $u = 2 + 3\mathrm{i}$ and $v = 3 + 2\mathrm{i}$. *[7 marks]*

(a) Given that $\frac{1}{u} + \frac{1}{v} = \frac{10}{w}$, express w in the form $a + b\mathrm{i},\ a,\ b \in \mathbb{R}$.

(b) Find w^* and express it in the form $re^{\mathrm{i}\theta}$.

Consider $w = 2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$

5a. Express w^2 and w^3 in modulus-argument form. *[3 marks]*

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5b. Sketch on an Argand diagram the points represented by w^0 , w^1 , w^2 and w^3. *[2 marks]*

These four points form the vertices of a quadrilateral, Q.

5c. Show that the area of the quadrilateral Q is $\frac{21\sqrt{3}}{2}$. *[3 marks]*

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5d. Let $z = 2\left(\cos\frac{\pi}{n} + \mathrm{i}\sin\frac{\pi}{n}\right),\ n \in \mathbb{Z}^+$. The points represented on an Argand diagram by $z^0,\ z^1,\ z^2,\ \ldots,\ z^n$ form the vertices of a polygon P_n. *[6 marks]*

Show that the area of the polygon P_n can be expressed in the form $a\left(b^n - 1\right)\sin\frac{\pi}{n}$, where $a,\ b \in \mathbb{R}$.