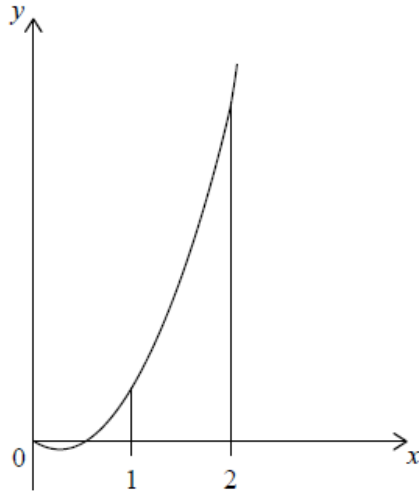


SL / Integration / Areas [62 marks]

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



1a. Find $\int (6x^2 - 3x) dx$.

[2 marks]

Markscheme

$$2x^3 - \frac{3x^2}{2} + c \quad \left(\text{accept } \frac{6x^3}{3} - \frac{3x^2}{2} + c \right) \quad \mathbf{A1A1 N2}$$

Notes: Award **A1A0** for both correct terms if $+c$ is omitted.

Award **A1A0** for one correct term *eg* $2x^3 + c$.

Award **A1A0** if both terms are correct, but candidate attempts further working to solve for c .

[2 marks]

1b. Find the area of the region enclosed by the graph of f , the x -axis and the lines $x = 1$ and $x = 2$. [4 marks]

Markscheme

substitution of limits or function **(A1)**

$$\text{eg } \int_1^2 f(x) \, dx, \left[2x^3 - \frac{3x^2}{2} \right]_1^2$$

substituting limits into their integrated function and subtracting **(M1)**

$$\text{eg } \frac{6 \times 2^3}{3} - \frac{3 \times 2^2}{2} - \left(\frac{6 \times 1^3}{3} + \frac{3 \times 1^2}{2} \right)$$

Note: Award **M0** if substituted into original function.

correct working **(A1)**

$$\text{eg } \frac{6 \times 8}{3} - \frac{3 \times 4}{2} - \frac{6 \times 1}{3} + \frac{3 \times 1}{2}, (16 - 6) - \left(2 - \frac{3}{2} \right)$$

$$\frac{19}{2} \quad \mathbf{A1 \, N3}$$

[4 marks]

Let $g(x) = -(x - 1)^2 + 5$.

2a. Write down the coordinates of the vertex of the graph of g .

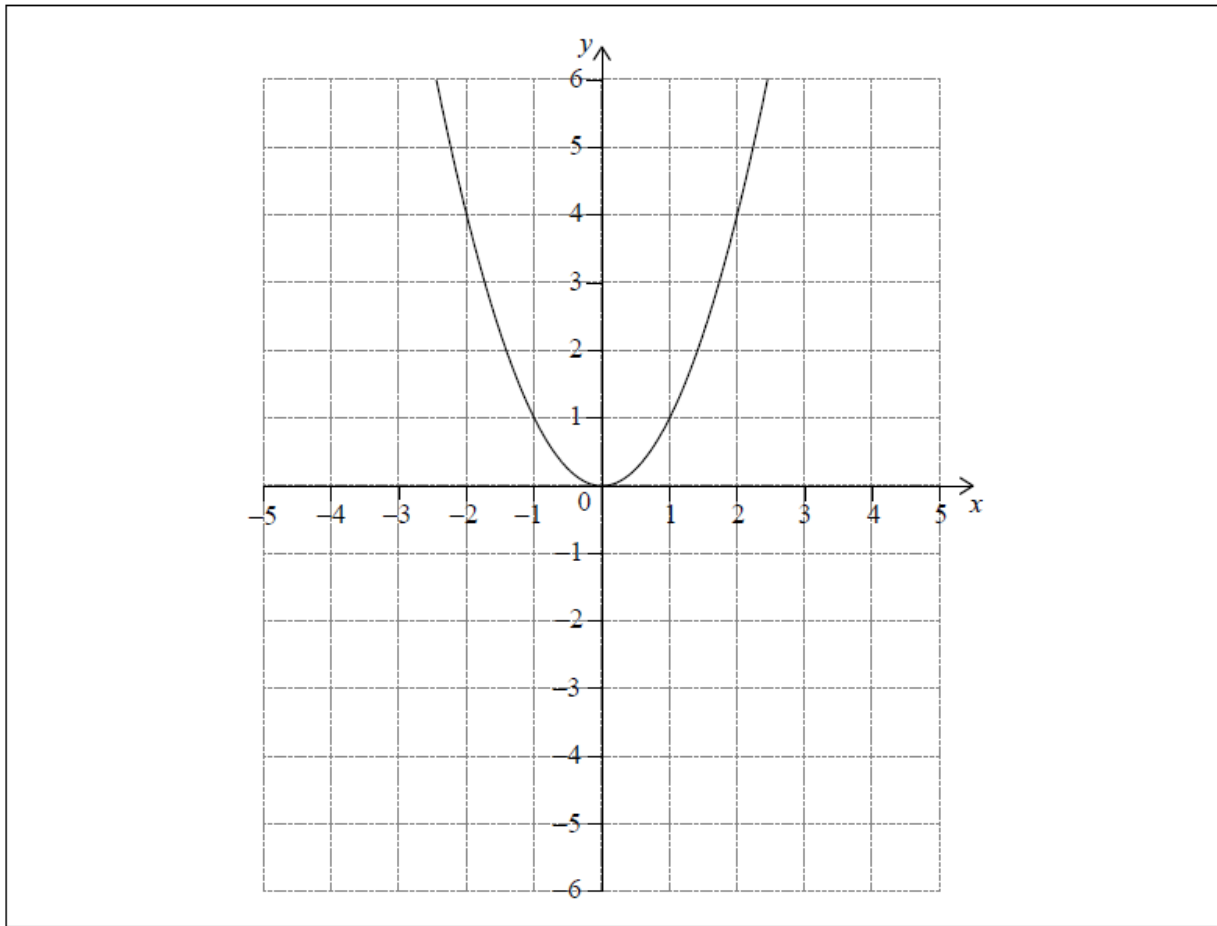
[1 mark]

Markscheme

(1,5) (exact) **A1 N1**

[1 mark]

Let $f(x) = x^2$. The following diagram shows part of the graph of f .

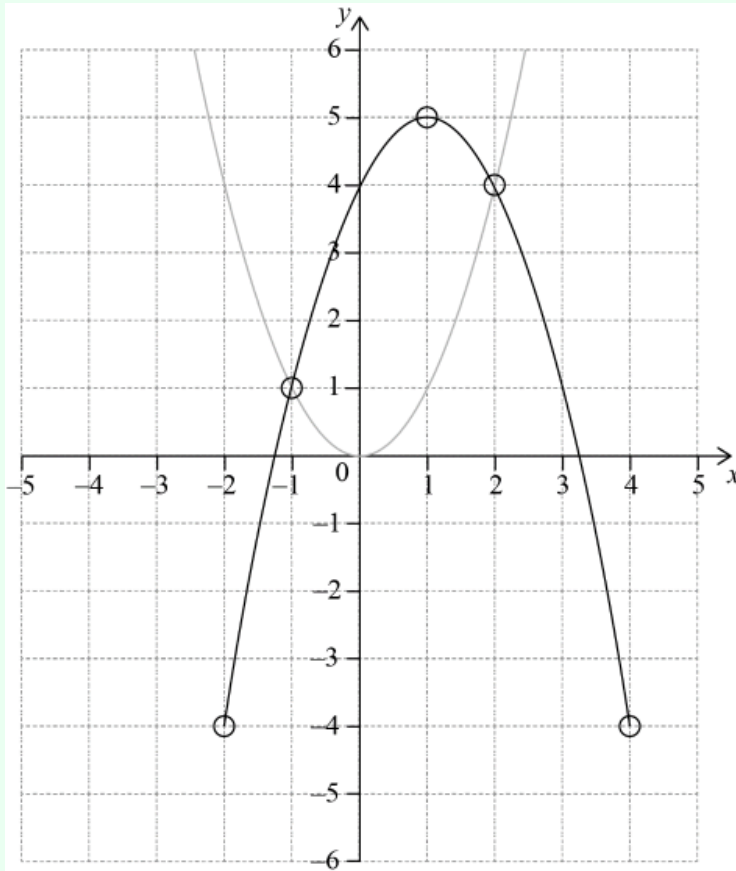


The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

2b. On the grid above, sketch the graph of g for $-2 \leq x \leq 4$.

[3 marks]

Markscheme



A1A1A1 N3

Note: The shape must be a concave-down parabola.
Only if the shape is correct, award the following for points in circles:
A1 for vertex,
A1 for correct intersection points,
A1 for correct endpoints.

[3 marks]

2c. Find the area of the region enclosed by the graphs of f and g .

[3 marks]

Markscheme

integrating and subtracting functions (in any order) **(M1)**

eg $\int f - g$

correct substitution of limits or functions (accept missing dx, but do not accept any errors, including extra bits) **(A1)**

eg $\int_{-1}^2 g - f$, $\int -(x - 1)^2 + 5 - x^2$

area = 9 (exact) **A1 N2**

[3 marks]

Let $f(x) = x^2$ and $g(x) = 3\ln(x + 1)$, for $x > -1$.

3a. Solve $f(x) = g(x)$.

[3 marks]

Markscheme

valid approach **(M1)**

eg sketch

0, 1.73843

$x = 0$, $x = 1.74$ (accept (0, 0) and (1.74, 3.02)) **A1A1 N3**

[3 marks]

3b. Find the area of the region enclosed by the graphs of f and g .

[3 marks]

Markscheme

integrating and subtracting functions (in any order) **(M1)**

eg $\int g - f$

correct substitution of their limits **or** function (accept missing dx)

(A1)

eg $\int_0^{1.74} g - f$, $\int 3 \ln(x + 1) - x^2$

Note: Do not award **A1** if there is an error in the substitution.

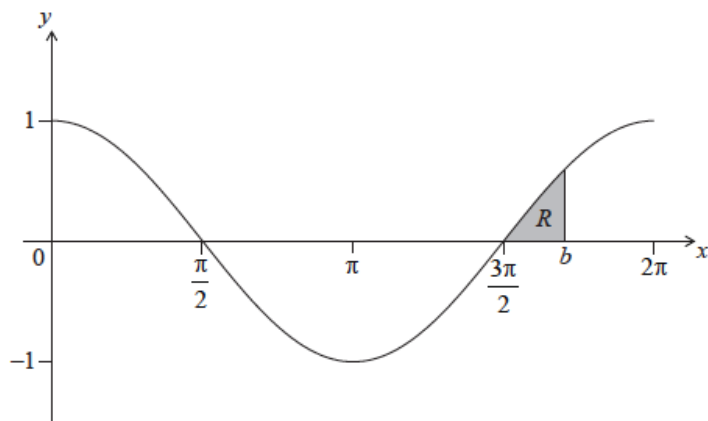
1.30940

1.31 **A1 N3**

[3 marks]

4. Let $f(x) = \cos x$, for $0 \leq x \leq 2\pi$. The following diagram shows the graph of f . **[8 marks]**

There are x -intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.



The shaded region R is enclosed by the graph of f , the line $x = b$, where $b > \frac{3\pi}{2}$, and the x -axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b .

Markscheme

attempt to set up integral (accept missing or incorrect limits and missing dx)
M1

eg $\int_{\frac{3\pi}{2}}^b \cos x dx$, $\int_a^b \cos x dx$, $\int_{\frac{3\pi}{2}}^b f dx$, $\int \cos x$

correct integration (accept missing or incorrect limits) **(A1)**

eg $[\sin x]_{\frac{3\pi}{2}}^b$, $\sin x$

substituting correct limits into **their** integrated function and subtracting (in any order) **(M1)**

eg $\sin b - \sin\left(\frac{3\pi}{2}\right)$, $\sin\left(\frac{3\pi}{2}\right) - \sin b$

$\sin\left(\frac{3\pi}{2}\right) = -1$ (seen anywhere) **(A1)**

setting **their** result from an integrated function equal to $\left(1 - \frac{\sqrt{3}}{2}\right)$ **M1**

eg $\sin b = -\frac{\sqrt{3}}{2}$

evaluating $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ **(A1)**

eg $b = \frac{\pi}{3}$, -60°

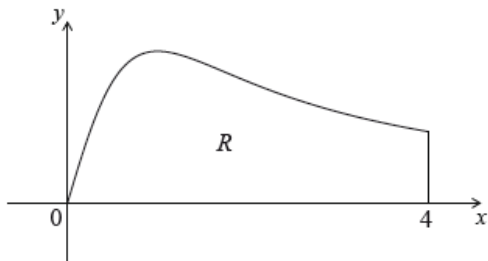
identifying correct value **(A1)**

eg $2\pi - \frac{\pi}{3}$, $360 - 60$

$b = \frac{5\pi}{3}$ **A1 N3**

[8 marks]

5. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, [6 marks] and the line $x = 4$.



Let R be the region enclosed by the graph of f , the x -axis and the line $x = 4$.
Find the area of R .

Markscheme

substitution of limits or function **(A1)**

eg $A = \int_0^4 f(x), \int \frac{x}{x^2+1} dx$

correct integration by substitution/inspection **A2**

$$\frac{1}{2} \ln(x^2 + 1)$$

substituting limits into **their** integrated function and subtracting (in any order)
(M1)

eg $\frac{1}{2}(\ln(4^2 + 1) - \ln(0^2 + 1))$

correct working **A1**

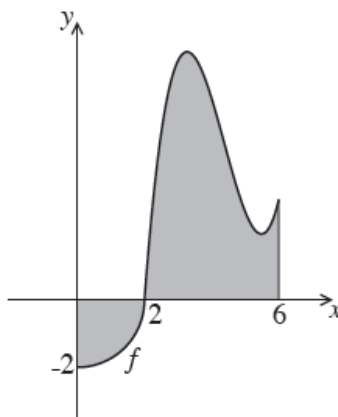
eg $\frac{1}{2}(\ln(4^2 + 1) - \ln(0^2 + 1)), \frac{1}{2}(\ln(17) - \ln(1)), \frac{1}{2} \ln 17 - 0$

$$A = \frac{1}{2} \ln(17) \quad \mathbf{A1} \quad \mathbf{N3}$$

Note: Exception to **FT** rule. Allow full **FT** on incorrect integration involving a \ln function.

[6 marks]

The following is the graph of a function f , for $0 \leq x \leq 6$.



The first part of the graph is a quarter circle of radius 2 with centre at the origin.

6a. Find $\int_0^2 f(x) dx$.

[4 marks]

Markscheme

attempt to find quarter circle area **(M1)**

eg $\frac{1}{4}(4\pi)$, $\frac{\pi r^2}{4}$, $\int_0^2 \sqrt{4-x^2} dx$

area of region = π **(A1)**

$$\int_0^2 f(x) dx = -\pi \quad \mathbf{A2} \quad \mathbf{N3}$$

[4 marks]

6b. The shaded region is enclosed by the graph of f , the x -axis, the y -axis **[3 marks]** and the line $x = 6$. The area of this region is 3π .

Find $\int_2^6 f(x) dx$.

Markscheme

attempted set up with both regions **(M1)**

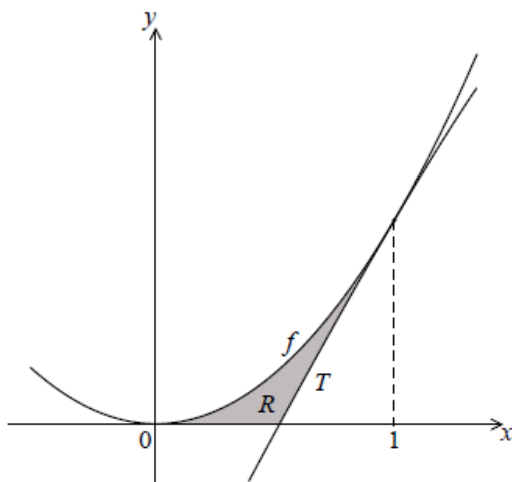
eg shaded area – quarter circle , $3\pi - \pi$, $3\pi - \int_0^2 f = \int_2^6 f$

$$\int_2^6 f(x) dx = 2\pi \quad \mathbf{A2} \quad \mathbf{N2}$$

[3 marks]

Total [7 marks]

The following diagram shows part of the graph of the function $f(x) = 2x^2$.



*diagram
not to scale*

The line T is the tangent to the graph of f at $x = 1$.

7a. Show that the equation of T is $y = 4x - 2$.

[5 marks]

Markscheme

$$f(1) = 2 \quad \mathbf{A1}$$

$$f'(x) = 4x \quad \mathbf{A1}$$

evidence of finding the gradient of f at $x = 1$ $\mathbf{M1}$

e.g. substituting $x = 1$ into $f'(x)$

finding gradient of f at $x = 1$ $\mathbf{A1}$

$$\text{e.g. } f'(1) = 4$$

evidence of finding equation of the line $\mathbf{M1}$

$$\text{e.g. } y - 2 = 4(x - 1), 2 = 4(1) + b$$

$$y = 4x - 2 \quad \mathbf{AG} \quad \mathbf{NO}$$

[5 marks]

7b. Find the x -intercept of T .

[2 marks]

Markscheme

appropriate approach **(M1)**

e.g. $4x - 2 = 0$

$$x = \frac{1}{2} \quad \mathbf{A1} \quad \mathbf{N2}$$

[2 marks]

- 7c. The shaded region R is enclosed by the graph of f , the line T , and the x - [9 marks]
axis.
- (i) Write down an expression for the area of R .
 - (ii) Find the area of R .

Markscheme

(i) bottom limit $x = 0$ (seen anywhere) **(A1)**

approach involving subtraction of integrals/areas **(M1)**

e.g. $\int f(x) - \text{area of triangle}$, $\int f - \int l$

correct expression **A2 N4**

e.g. $\int_0^1 2x^2 dx - \int_{0.5}^1 (4x - 2) dx$, $\int_0^1 f(x) dx - \frac{1}{2}$,
 $\int_0^{0.5} 2x^2 dx + \int_{0.5}^1 (f(x) - (4x - 2)) dx$

(ii) **METHOD 1 (using only integrals)**

correct integration **(A1)(A1)(A1)**

$\int 2x^2 dx = \frac{2x^3}{3}$, $\int (4x - 2) dx = 2x^2 - 2x$

substitution of limits **(M1)**

e.g. $\frac{1}{12} + \frac{2}{3} - 2 + 2 - \left(\frac{1}{12} - \frac{1}{2} + 1\right)$

area = $\frac{1}{6}$ **A1 N4**

METHOD 2 (using integral and triangle)

area of triangle = $\frac{1}{2}$ **(A1)**

correct integration **(A1)**

$\int 2x^2 dx = \frac{2x^3}{3}$

substitution of limits **(M1)**

e.g. $\frac{2}{3}(1)^3 - \frac{2}{3}(0)^3$, $\frac{2}{3} - 0$

correct simplification **(A1)**

e.g. $\frac{2}{3} - \frac{1}{2}$

area = $\frac{1}{6}$ **A1 N4**

[9 marks]

8. Let $f(x) = \cos(x^2)$ and $g(x) = e^x$, for $-1.5 \leq x \leq 0.5$.

[6 marks]

Find the area of the region enclosed by the graphs of f and g .

Markscheme

evidence of finding intersection points **(M1)**

e.g. $f(x) = g(x)$, $\cos x^2 = e^x$, sketch showing intersection

$x = -1.11$, $x = 0$ (may be seen as limits in the integral) **A1A1**

evidence of approach involving integration and subtraction (in any order)
(M1)

e.g. $\int_{-1.11}^0 \cos x^2 - e^x$, $\int(\cos x^2 - e^x)dx$, $\int g - f$

area = 0.282 **A2 N3**

[6 marks]