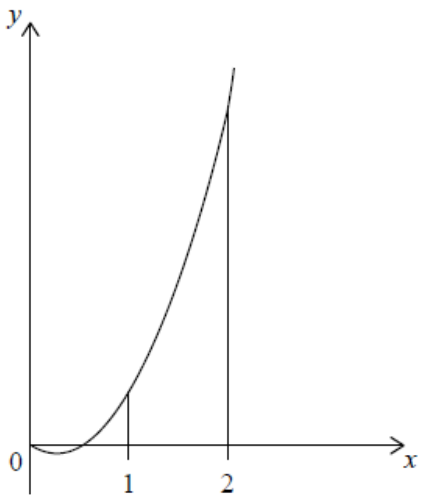
SL / Integration / Areas *[62 marks]*

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



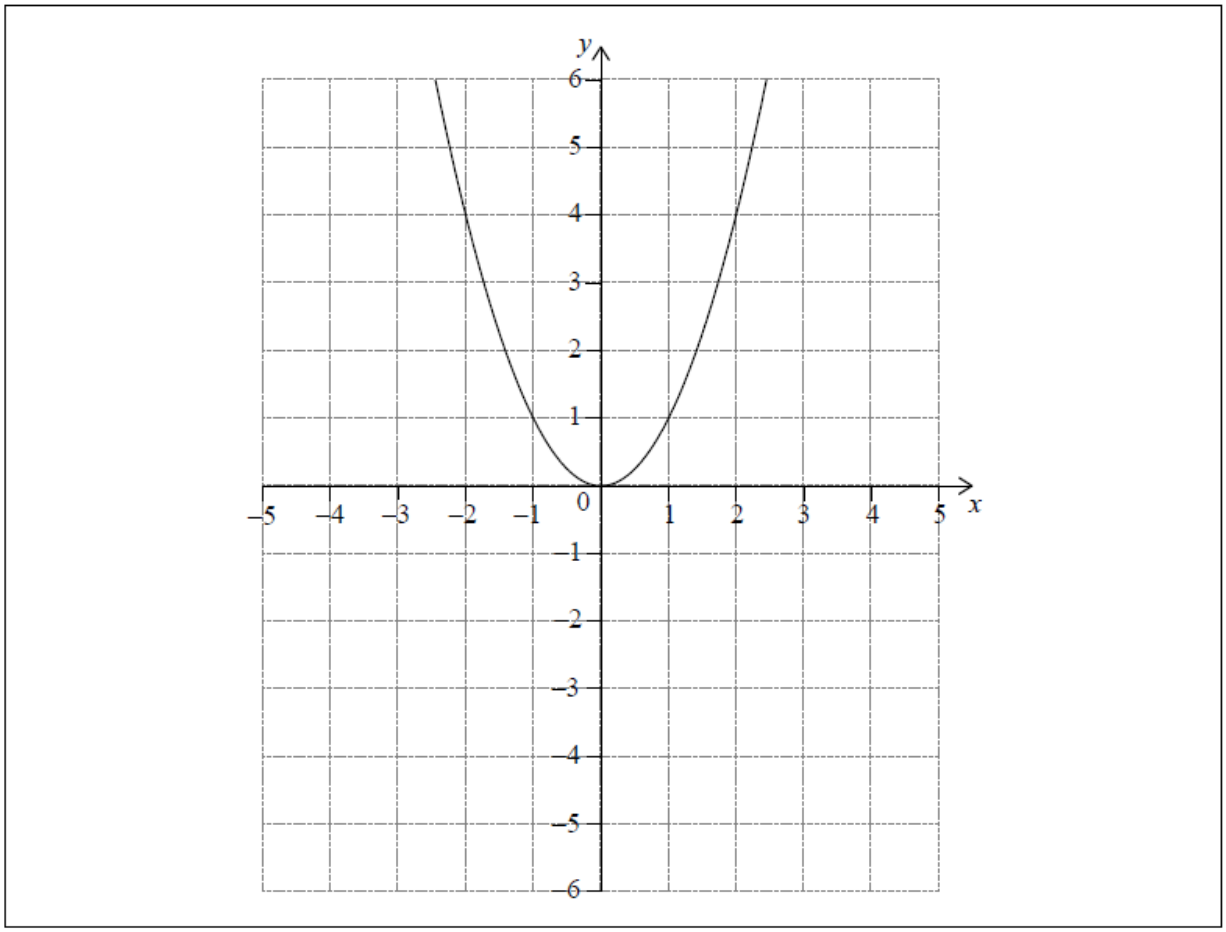
1a. Find $\int (6x^2 - 3x)\,\mathrm{d}x$. *[2 marks]*

1b. Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. *[4 marks]*

Let $g(x) = -(x - 1)^2 + 5$.

2a. Write down the coordinates of the vertex of the graph of g. *[1 mark]*

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

2b. On the grid above, sketch the graph of g for $-2 \leq x \leq 4$. *[3 marks]*

2c. Find the area of the region enclosed by the graphs of f and g. *[3 marks]*

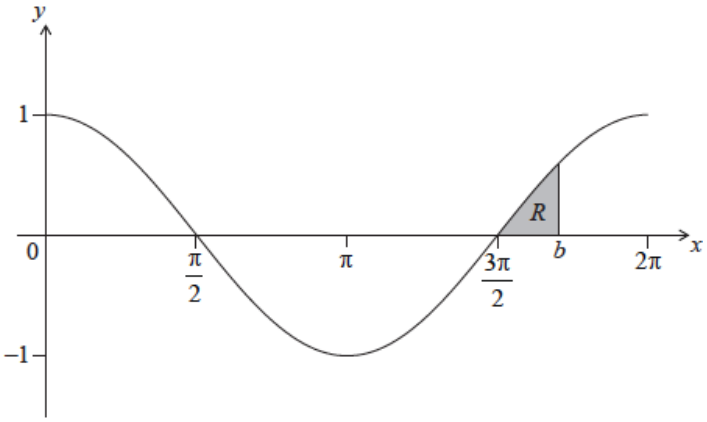
Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

3a. Solve $f(x) = g(x)$. *[3 marks]*

3b. Find the area of the region enclosed by the graphs of f and g. *[3 marks]*

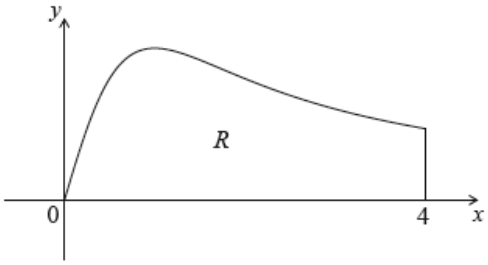
4. Let $f(x) = \cos x$, for $0 \leq x \leq 2\pi$. The following diagram shows the graph of f. *[8 marks]*

There are x-intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.



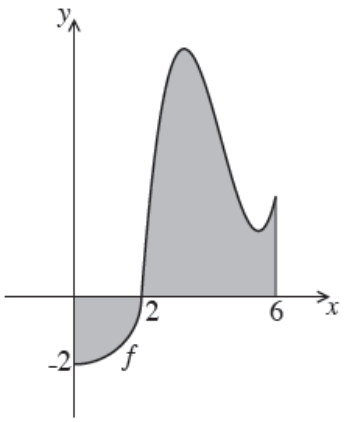
The shaded region R is enclosed by the graph of f, the line $x = b$, where $b > \frac{3\pi}{2}$, and the x-axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b.

5. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, and the line $x = 4$. *[6 marks]*



Let R be the region enclosed by the graph of f , the x-axis and the line $x = 4$. Find the area of R.

The following is the graph of a function f, for $0 \leq x \leq 6$.



The first part of the graph is a quarter circle of radius 2 with centre at the origin.

6a. Find $\int_0^2 f(x)\mathrm{d}x$. *[4 marks]*

6b. The shaded region is enclosed by the graph of f, the x-axis, the y-axis and the line $x = 6$. The area of this region is 3π. *[3 marks]*

Find $\int_2^6 f(x)\mathrm{d}x$.

The following diagram shows part of the graph of the function $f(x) = 2x^2$.

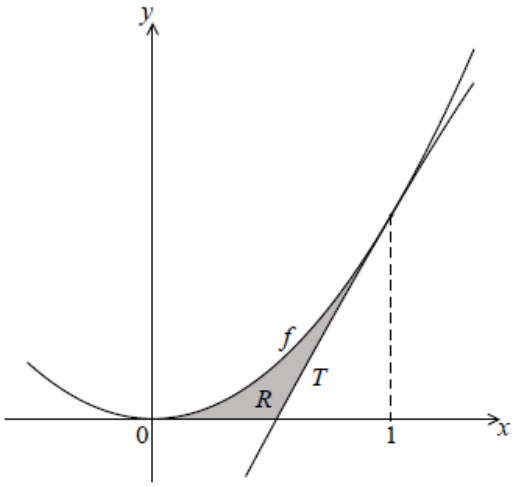


diagram not to scale

The line *T* is the tangent to the graph of *f* at $x = 1$.

7a. Show that the equation of *T* is $y = 4x - 2$. *[5 marks]*

7b. Find the *x*-intercept of *T*. *[2 marks]*

7c. The shaded region R is enclosed by the graph of f, the line T, and the x-axis. *[9 marks]*

(i) Write down an expression for the area of R.

(ii) Find the area of R.

8. Let $f(x) = \cos(x^2)$ and $g(x) = e^x$, for $-1.5 \leq x \leq 0.5$. *[6 marks]*

Find the area of the region enclosed by the graphs of f and g.