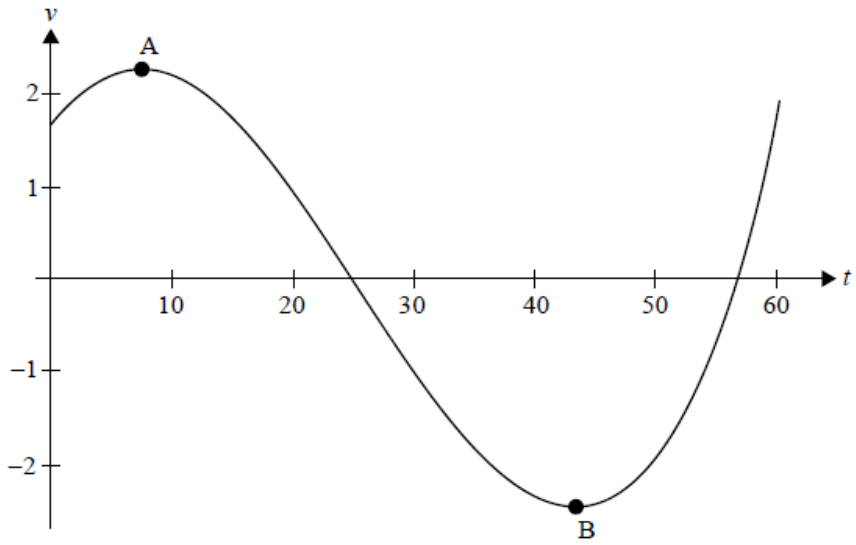
HL / velocity *[52 marks]*

A body moves in a straight line such that its velocity, $v\,\text{ms}^{-1}$, after t seconds is given by $v = 2\sin\left(\frac{t}{10} + \frac{\pi}{5}\right)\csc\left(\frac{t}{30} + \frac{\pi}{4}\right)$ for $0 \leqslant t \leqslant 60$.

The following diagram shows the graph of v against t. Point A is a local maximum and point B is a local minimum.



1a. Determine the coordinates of point A and the coordinates of point B. *[4 marks]*

1b. Hence, write down the maximum speed of the body. *[1 mark]*

The body first comes to rest at time $t = t_1$. Find

1c. the value of t_1. *[2 marks]*

1d. the distance travelled between $t = 0$ and $t = t_1$. *[2 marks]*

1e. the acceleration when $t = t_1$. *[2 marks]*

1f. Find the distance travelled in the first 30 seconds. *[3 marks]*

A point P moves in a straight line with velocity v ms^{-1} given by $v(t) = \mathrm{e}^{-t} - 8t^2\mathrm{e}^{-2t}$ at time t seconds, where $t \geq 0$.

2a. Determine the first time t_1 at which P has zero velocity. *[2 marks]*

2b. Find an expression for the acceleration of P at time t. *[2 marks]*

2c. Find the value of the acceleration of P at time t_1. *[1 mark]*

3. A particle moves in a straight line such that at time t seconds $(t \geqslant 0)$, its velocity v, in ms^{-1}, is given by $v = 10t\mathrm{e}^{-2t}$. Find the exact distance travelled by the particle in the first half-second. *[5 marks]*

The displacement, s, in metres, of a particle t seconds after it passes through the origin is given by the expression $s = \ln(2 - e^{-t}), t \geqslant 0$.

4a. Find an expression for the velocity, v, of the particle at time t. *[2 marks]*

4b. Find an expression for the acceleration, a, of the particle at time t. *[2 marks]*

4c. Find the acceleration of the particle at time $t = 0$. *[1 mark]*

A particle can move along a straight line from a point O. The velocity v, in ms^{-1}, is given by the function $v(t) = 1 - \mathrm{e}^{-\sin t^2}$ where time $t \geq 0$ is measured in seconds.

5a. Write down the first two times $t_1,\ t_2 > 0$, when the particle changes direction. *[2 marks]*

5b. (i) Find the time $t < t_2$ when the particle has a maximum velocity. *[4 marks]*

(ii) Find the time $t < t_2$ when the particle has a minimum velocity.

5c. Find the distance travelled by the particle between times $t = t_1$ and $t = t_2$. *[2 marks]*

A particle moves in a straight line, its velocity $v \text{ms}^{-1}$ at time t seconds is given by $v = 9t - 3t^2, 0 \leq t \leq 5$.

At time $t = 0$, the displacement s of the particle from an origin O is 3 m.

6a. Find the displacement of the particle when $t = 4$. *[3 marks]*

6b. Sketch a displacement/time graph for the particle, $0 \leq t \leq 5$, showing clearly where the curve meets the axes and the coordinates of the points where the displacement takes greatest and least values. *[5 marks]*

6c. For $t > 5$, the displacement of the particle is given by $s = a + b\cos\frac{2\pi t}{5}$ such that s is continuous for all $t \geq 0$. *[3 marks]*

Given further that $s = 16.5$ when $t = 7.5$, find the values of a and b.

6d. For $t > 5$, the displacement of the particle is given by $s = a + b\cos\frac{2\pi t}{5}$ such that s is continuous for all $t \geq 0$. *[4 marks]*

Find the times t_1 and $t_2 (0 < t_1 < t_2 < 8)$ when the particle returns to its starting point.