HL / Integration 2 *[131 marks]*

1. Find $\int \arcsin x \, dx$ *[5 marks]*

Markscheme

attempt at integration by parts with $u = \arcsin x$ and $v' = 1$ ***M1***

$\int \arcsin x \, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$ ***A1A1***

Note: Award ***A1*** for $x \arcsin x$ and ***A1*** for $-\int \frac{x}{\sqrt{1-x^2}} dx$.

solving $\int \frac{x}{\sqrt{1-x^2}} dx$ by substitution with $u = 1 - x^2$ or inspection ***(M1)***

$\int \arcsin x dx = x \arcsin x + \sqrt{1 - x^2} + c$ ***A1***

[5 marks]

2a. Using the substitution $x = \tan\theta$ show that $\int_0^1 \frac{1}{(x^2+1)^2} dx = \int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$. *[4 marks]*

Markscheme

let $x = \tan\theta$

$\Rightarrow \frac{dx}{d\theta} = \sec^2\theta$ ***(A1)***

$\int \frac{1}{(x^2+1)^2} dx = \int \frac{\sec^2\theta}{(\tan^2\theta+1)^2} d\theta$ ***M1***

Note: The method mark is for an attempt to substitute for both x and dx.

$= \int \frac{1}{\sec^2\theta} d\theta$ (or equivalent) ***A1***

when $x = 0,\ \theta = 0$ and when $x = 1,\ \theta = \frac{\pi}{4}$ ***M1***

$\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$ ***AG***

[4 marks]

2b. Hence find the value of $\int_0^1 \frac{1}{(x^2+1)^2} dx$. *[3 marks]*

Markscheme

$\left(\int_0^1 \frac{1}{(x^2+1)^2}\mathrm{d}x = \int_0^{\frac{\pi}{4}} \cos^2\theta \mathrm{d}\theta\right) = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1+\cos 2\theta)\,\mathrm{d}\theta$ ***M1***

$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$ ***A1***

$= \frac{\pi}{8} + \frac{1}{4}$ ***A1***

[3 marks]

3a. Find $\int (1+\tan^2 x)\mathrm{d}x$. *[2 marks]*

Markscheme

$\int (1+\tan^2 x)\mathrm{d}x = \int \sec^2 x \mathrm{d}x = \tan x (+c)$ ***M1A1***

[2 marks]

3b. Find $\int \sin^2 x \mathrm{d}x$. *[3 marks]*

Markscheme

$\int \sin^2 x \mathrm{d}x = \int \frac{1-\cos 2x}{2}\mathrm{d}x$ ***M1A1***

$= \frac{x}{2} - \frac{\sin 2x}{4}(+c)$ ***A1***

Note: Allow integration by parts followed by trig identity.

Award ***M1*** for parts, ***A1*** for trig identity, ***A1*** final answer.

[3 marks]

Total [5 marks]

---

4. By using the substitution $t = \tan x$, find $\int \frac{\mathrm{d}x}{1+\sin^2 x}$. *[8 marks]*

Express your answer in the form $m \arctan(n \tan x) + c$, where m, n are constants to be determined.

Markscheme

EITHER

$x = \arctan t$ ***(M1)***

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{1+t^2}$ ***A1***

OR

$t = \tan x$

$\frac{\mathrm{d}t}{\mathrm{d}x} = \sec^2 x$ ***(M1)***

$= 1 + \tan^2 x$ ***A1***

$= 1 + t^2$

THEN

$\sin x = \frac{t}{\sqrt{1+t^2}}$ ***(A1)***

Note: This ***A1*** is independent of the first two marks

$\int \frac{\mathrm{d}x}{1+\sin^2 x} = \int \frac{\frac{\mathrm{d}t}{1+t^2}}{1+\left(\frac{t}{\sqrt{1+t^2}}\right)^2}$ ***M1A1***

Note: Award ***M1*** for attempting to obtain integral in terms of t and $\mathrm{d}t$

$= \int \frac{\mathrm{d}t}{(1+t^2)+t^2} = \int \frac{\mathrm{d}t}{1+2t^2}$ ***A1***

$= \frac{1}{2}\int \frac{\mathrm{d}t}{\frac{1}{2}+t^2} = \frac{1}{2} \times \frac{1}{\frac{1}{\sqrt{2}}}\arctan\left(\frac{t}{\frac{1}{\sqrt{2}}}\right)$ ***A1***

$= \frac{\sqrt{2}}{2}\arctan\left(\sqrt{2}\tan x\right)(+c)$ ***A1***

[8 marks]

5. Use the substitution $x = a\sec\theta$ to show that $\int_{a\sqrt{2}}^{2a} \frac{\mathrm{d}x}{x^3\sqrt{x^2-a^2}} = \frac{1}{24a^3}\left(3\sqrt{3}+\pi-6\right)$. *[7 marks]*

Markscheme

$x = a\sec\theta$

$\frac{dx}{d\theta} = a\sec\theta\tan\theta$ ***(A1)***

new limits:

$x = a\sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$ and
$x = 2a \Rightarrow \theta = \frac{\pi}{3}$ ***(A1)***

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{a\sec\theta\tan\theta}{a^3\sec^3\theta\sqrt{a^2\sec^2\theta - a^2}} d\theta$ ***M1***

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2\theta}{a^3} d\theta$ ***A1***

using
$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ ***M1***

$\frac{1}{2a^3}\left[\frac{1}{2}\sin 2\theta + \theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ or equivalent ***A1***

$= \frac{1}{4a^3}\left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3} - 1 - \frac{\pi}{2}\right)$ or equivalent ***A1***

$= \frac{1}{24a^3}\left(3\sqrt{3} + \pi - 6\right)$ ***AG***

[7 marks]

---

6. A particle moves in a straight line such that at time t seconds $(t \geqslant 0)$, its velocity v, in ms^{-1}, is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second. *[5 marks]*

Markscheme

$s = \int_0^{\frac{1}{2}} 10te^{-2t}dt$

attempt at integration by parts ***M1***

$= \left[-5te^{-2t}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} -5e^{-2t}dt$ ***A1***

$= \left[-5te^{-2t} - \frac{5}{2}e^{-2t}\right]_0^{\frac{1}{2}}$ ***(A1)***

Note: Condone absence of limits (or incorrect limits) and missing factor of 10 up to this point.

$s = \int_0^{\frac{1}{2}} 10te^{-2t}dt$ ***(M1)***

$= -5e^{-1} + \frac{5}{2}\ \left(= \frac{-5}{e} + \frac{5}{2}\right)\ \left(= \frac{5e-10}{2e}\right)$ ***A1***

[5 marks]

7a. Express $x^2 + 3x + 2$ in the form $(x + h)^2 + k$. *[1 mark]*

Markscheme

$x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$ ***A1***

[1 mark]

7b. Factorize x^2+3x+2. *[1 mark]*

Markscheme

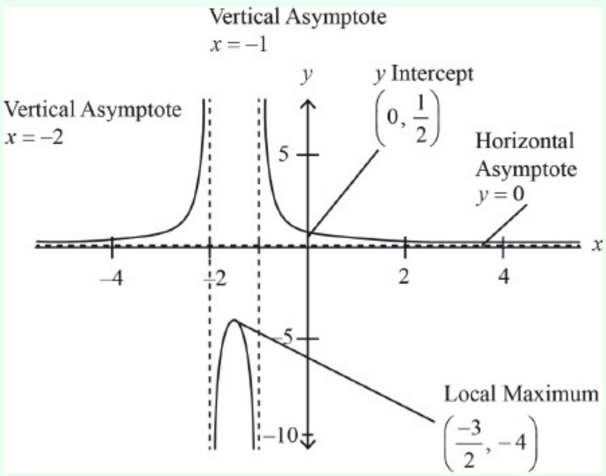
$x^2+3x+2=(x+2)(x+1)$ ***A1***

[1 mark]

Consider the function $f(x)=\frac{1}{x^2+3x+2},\ x\in\mathbb{R},\ x\neq-2,\ x\neq-1$.

7c. Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y-intercept and the local maximum. *[5 marks]*

Markscheme



A1 for the shape

A1 for the equation $y=0$

A1 for asymptotes $x=-2$ and $x=-1$

A1 for coordinates $\left(-\frac{3}{2},\ -4\right)$

A1 y-intercept $\left(0,\ \frac{1}{2}\right)$

[5 marks]

7d. Show that $\frac{1}{x+1}-\frac{1}{x+2}=\frac{1}{x^2+3x+2}$. *[1 mark]*

Markscheme

$\frac{1}{x+1}-\frac{1}{x+2}=\frac{(x+2)-(x+1)}{(x+1)(x+2)}$ ***M1***

$=\frac{1}{x^2+3x+2}$ ***AG***

[1 mark]

7e. Hence find the value of p if $\int_0^1 f(x)\mathrm{d}x=\ln(p)$. *[4 marks]*

Markscheme

$\int_0^1 \frac{1}{x+1} - \frac{1}{x+2} \mathrm{d}x$

$= [\ln(x+1) - \ln(x+2)]_0^1$ ***A1***

$= \ln 2 - \ln 3 - \ln 1 + \ln 2$ ***M1***

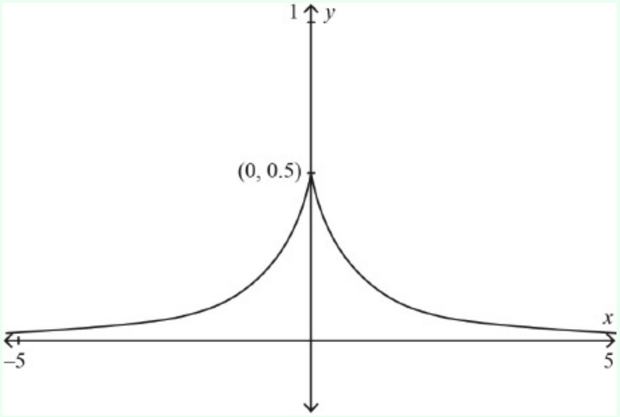
$= \ln\left(\frac{4}{3}\right)$ ***M1A1***

$\therefore p = \frac{4}{3}$

[4 marks]

7f. Sketch the graph of $y = f(|x|)$. *[2 marks]*

Markscheme



symmetry about the y-axis ***M1***

correct shape ***A1***

Note: Allow ***FT*** from part (b).

[2 marks]

7g. Determine the area of the region enclosed between the graph of $y = f(|x|)$, the x-axis and the lines with equations $x = -1$ and $x = 1$. *[3 marks]*

Markscheme

$2\int_0^1 f(x)\mathrm{d}x$ ***(M1)(A1)***

$= 2\ln\left(\frac{4}{3}\right)$ ***A1***

Note: Do not award ***FT*** from part (e).

[3 marks]

A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t,\ t \geqslant 0$. The first two times when the particle is at rest are denoted by t_1 and t_2, where $t_1 < t_2$.

8a. Find t_1 and t_2. *[5 marks]*

Markscheme

$s = t + \cos 2t$

$\frac{\mathrm{d}s}{\mathrm{d}t} = 1 - 2\sin 2t$ ***M1A1***

$= 0$ ***M1***

$\Rightarrow \sin 2t = \frac{1}{2}$

$t_1 = \frac{\pi}{12}(s),\ t_2 = \frac{5\pi}{12}(s)$ ***A1A1***

Note: Award ***A0A0*** if answers are given in degrees.

[5 marks]

8b. Find the displacement of the particle when $t = t_1$ *[2 marks]*

Markscheme

$s = \frac{\pi}{12} + \cos\frac{\pi}{6}\ \left(s = \frac{\pi}{12} + \frac{\sqrt{3}}{2}(m)\right)$ ***A1A1***

[2 marks]

Consider the curve defined by the equation $4x^2 + y^2 = 7$.

9a. Find the equation of the normal to the curve at the point $\left(1,\ \sqrt{3}\right)$. *[6 marks]*

Markscheme

METHOD 1

$4x^2 + y^2 = 7$

$8x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***(M1)(A1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4x}{y}$

Note: Award ***M1A1*** for finding $\frac{\mathrm{d}y}{\mathrm{d}x} = -2.309\ldots$ using any alternative method.

hence gradient of normal $= \frac{y}{4x}$ ***(M1)***

hence gradient of normal at $\left(1, \sqrt{3}\right)$ is $\frac{\sqrt{3}}{4}$ $(= 0.433)$ ***(A1)***

hence equation of normal is $y - \sqrt{3} = \frac{\sqrt{3}}{4}(x - 1)$ ***(M1)A1***

$\left(y = \frac{\sqrt{3}}{4}x + \frac{3\sqrt{3}}{4}\right)$ $(y = 0.433x + 1.30)$

METHOD 2

$4x^2 + y^2 = 7$

$y = \sqrt{7 - 4x^2}$ ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4x}{\sqrt{7-4x^2}}$ ***(A1)***

Note: Award ***M1A1*** for finding $\frac{\mathrm{d}y}{\mathrm{d}x} = -2.309\ldots$ using any alternative method.

hence gradient of normal $= \frac{\sqrt{7-4x^2}}{4x}$ ***(M1)***

hence gradient of normal at $\left(1, \sqrt{3}\right)$ is $\frac{\sqrt{3}}{4}$ $(= 0.433)$ ***(A1)***

hence equation of normal is $y - \sqrt{3} = \frac{\sqrt{3}}{4}(x - 1)$ ***(M1)A1***

$\left(y = \frac{\sqrt{3}}{4}x + \frac{3\sqrt{3}}{4}\right)$ $(y = 0.433x + 1.30)$

[6 marks]

9b. Find the volume of the solid formed when the region bounded by the curve, the x-axis for $x \geqslant 0$ and the y-axis for $y \geqslant 0$ is rotated through 2π about the x-axis. *[3 marks]*

Markscheme

Use of $V = \pi\int_0^{\frac{\sqrt{7}}{2}} y^2 \mathrm{d}x$

$V = \pi\int_0^{\frac{\sqrt{7}}{2}} \left(7 - 4x^2\right)\mathrm{d}x$ ***(M1)(A1)***

Note: Condone absence of limits or incorrect limits for ***M*** mark.

Do not condone absence of or multiples of π.

$= 19.4$ $\left(= \frac{7\sqrt{7}\pi}{3}\right)$ ***A1***

[3 marks]

Let the function f be defined by $f(x) = \frac{2-e^x}{2e^x-1}$, $x \in D$.

10a. Determine D, the largest possible domain of f. *[2 marks]*

Markscheme

attempting to solve either $2e^x - 1 = 0$ or $2e^x - 1 \neq 0$ for x ***(M1)***

$D = \mathbb{R} \setminus \{-\ln 2\}$ (or equivalent *eg* $x \neq -\ln 2$) ***A1***

Note: Accept $D = \mathbb{R} \setminus \{-0.693\}$ or equivalent *eg* $x \neq -0.693$.

[2 marks]

10b. Show that the graph of f has three asymptotes and state their equations. *[5 marks]*

Markscheme

considering $\lim\limits_{x \to -\ln 2} f(x)$ ***(M1)***

$x = -\ln 2$ $(x = -0.693)$ ***A1***

considering one of $\lim\limits_{x \to -\infty} f(x)$ or $\lim\limits_{x \to +\infty} f(x)$ ***M1***

$\lim\limits_{x \to -\infty} f(x) = -2 \Rightarrow y = -2$ ***A1***

$\lim\limits_{x \to +\infty} f(x) = -\frac{1}{2} \Rightarrow y = -\frac{1}{2}$ ***A1***

Note: Award ***A0A0*** for $y = -2$ and $y = -\frac{1}{2}$ stated without any justification.

[5 marks]

10c. Show that $f'(x) = -\frac{3e^x}{(2e^x-1)^2}$. *[3 marks]*

Markscheme

$f'(x) = \frac{-e^x(2e^x-1) - 2e^x(2-e^x)}{(2e^x-1)^2}$ ***M1A1A1***

$= -\frac{3e^x}{(2e^x-1)^2}$ ***AG***

[3 marks]

10d. Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. *[4 marks]*

Markscheme

$f'(x) < 0$ (for all $x \in D$) $\Rightarrow f$ is (strictly) decreasing ***R1***

Note: Award ***R1*** for a statement such as $f'(x) \neq 0$ and so the graph of f has no turning points.

one branch is above the upper horizontal asymptote and the other branch is below the lower horizontal asymptote ***R1***

f has an inverse ***AG***

$-\infty < x < -2 \cup -\frac{1}{2} < x < \infty$ ***A2***

Note: Award ***A2*** if the domain of the inverse is seen in either part (d) or in part (e).

[4 marks]

10e. Find an expression for $f^{-1}(x)$. *[4 marks]*

Markscheme

$x = \frac{2-\mathrm{e}^y}{2\mathrm{e}^y-1}$ ***M1***

Note: Award ***M1*** for interchanging x and y (can be done at a later stage).

$2x\mathrm{e}^y - x = 2 - \mathrm{e}^y$ ***M1***

$\mathrm{e}^y(2x+1) = x+2$ ***A1***

$f^{-1}(x) = \ln\left(\frac{x+2}{2x+1}\right)$ $\left(f^{-1}(x) = \ln(x+2) - \ln(2x+1)\right)$ ***A1***

[4 marks]

10f. Consider the region R enclosed by the graph of $y = f(x)$ and the axes. *[4 marks]*

Find the volume of the solid obtained when R is rotated through 2π about the y-axis.

Markscheme

use of $V = \pi \int_a^b x^2 \mathrm{d}y$ ***(M1)***

$= \pi \int_0^1 \left(\ln\left(\frac{y+2}{2y+1}\right)\right)^2 \mathrm{d}y$ ***(A1)(A1)***

Note: Award ***(A1)*** for the correct integrand and ***(A1)*** for the limits.

$= 0.331$ ***A1***

[4 marks]

Consider the function defined by $f(x) = x\sqrt{1-x^2}$ on the domain $-1 \leq x \leq 1$.

11a. Show that f is an odd function. *[2 marks]*

Markscheme

$f(-x) = (-x)\sqrt{1-(-x)^2}$ ***M1***

$= -x\sqrt{1-x^2}$

$= -f(x)$ ***R1***

hence f is odd ***AG***

[2 marks]

11b. Find $f'(x)$. *[3 marks]*

Markscheme

$f'(x) = x \bullet \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \bullet -2x + (1-x^2)^{\frac{1}{2}}$ ***M1A1A1***

[3 marks]

11c. Hence find the x-coordinates of any local maximum or minimum points. *[3 marks]*

Markscheme

$f'(x) = \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$ $\left(= \frac{1-2x^2}{\sqrt{1-x^2}}\right)$ ***A1***

Note: This may be seen in part (b).

Note: Do not allow FT from part (b).

$f'(x) = 0 \Rightarrow 1 - 2x^2 = 0$ ***M1***

$x = \pm\frac{1}{\sqrt{2}}$ ***A1***

[3 marks]

11d. Find the range of f. *[3 marks]*

Markscheme

y-coordinates of the Max Min Points are $y = \pm\frac{1}{2}$ ***M1A1***

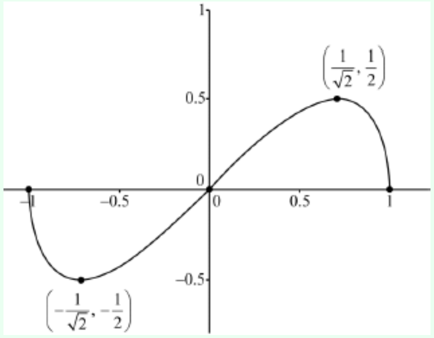
so range of $f(x)$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$ ***A1***

Note: Allow FT from (c) if values of x, within the domain, are used.

[3 marks]

11e. Sketch the graph of $y = f(x)$ indicating clearly the coordinates of the x-intercepts and any local maximum or minimum points. *[3 marks]*

Markscheme



Shape: The graph of an odd function, on the given domain, s-shaped,

where the max(min) is the right(left) of 0.5 (-0.5) ***A1***

x-intercepts ***A1***

turning points ***A1***

[3 marks]

11f. Find the area of the region enclosed by the graph of $y = f(x)$ and the x-axis for $x \geq 0$. *[4 marks]*

Markscheme

$\text{area} = \int_0^1 x\sqrt{1-x^2}\text{d}x$ ***(M1)***

attempt at "backwards chain rule" or substitution ***M1***

$= -\frac{1}{2}\int_0^1 (-2x)\sqrt{1-x^2}\text{d}x$

Note: Condone absence of limits for first two marks.

$= \left[\frac{2}{3}(1-x^2)^{\frac{3}{2}} \bullet -\frac{1}{2}\right]_0^1$ ***A1***

$= \left[-\frac{1}{3}(1-x^2)^{\frac{3}{2}}\right]_0^1$

$= 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$ ***A1***

[4 marks]

11g. Show that $\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\text{d}x > \left|\int_{-1}^{1} x\sqrt{1-x^2}\text{d}x\right|$. *[2 marks]*

Markscheme

$\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\text{d}x > 0$ ***R1***

$\left|\int_{-1}^{1} x\sqrt{1-x^2}\text{d}x\right| = 0$ ***R1***

so $\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\text{d}x > \left|\int_{-1}^{1} x\sqrt{1-x^2}\text{d}x\right|$ ***AG***

[2 marks]

Total [20 marks]

12a. Express $4x^2 - 4x + 5$ in the form $a(x - h)^2 + k$ where *a*, *h*, $k \in \mathbb{Q}$. *[2 marks]*

Markscheme

$4(x - 0.5)^2 + 4$ ***A1A1***

Note: ***A1*** for two correct parameters, ***A2*** for all three correct.

[2 marks]

12b. The graph of $y = x^2$ is transformed onto the graph of $y = 4x^2 - 4x + 5$. Describe a sequence of transformations that does this, making the order of transformations clear. *[3 marks]*

Markscheme

translation $\begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$ (allow "0.5 to the right") ***A1***

stretch parallel to *y*-axis, scale factor 4 (allow vertical stretch or similar) ***A1***

translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ (allow "4 up") ***A1***

Note: All transformations must state magnitude and direction.

Note: First two transformations can be in either order.

It could be a stretch followed by a single translation of $\begin{pmatrix} 0.5 \\ 4 \end{pmatrix}$. If the vertical translation is before the stretch it is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

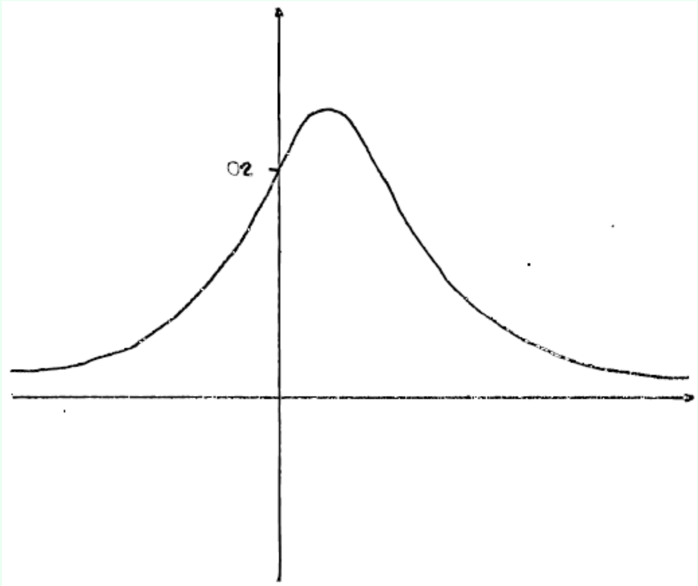
[3 marks]

The function *f* is defined by

$f(x) = \frac{1}{4x^2 - 4x + 5}$.

12c. Sketch the graph of $y = f(x)$. *[2 marks]*

Markscheme



general shape (including asymptote and single maximum in first quadrant), ***A1***

intercept
$\left(0, \frac{1}{5}\right)$ or maximum
$\left(\frac{1}{2}, \frac{1}{4}\right)$ shown ***A1***

[2 marks]

12d. Find the range of *f*. *[2 marks]*

Markscheme

$0 < f(x) \leqslant \frac{1}{4}$ ***A1A1***

Note: ***A1*** for
$\leqslant \frac{1}{4}$, ***A1*** for
$0 <$.

[2 marks]

12e. By using a suitable substitution show that
$\int f(x)\mathrm{d}x = \frac{1}{4}\int \frac{1}{u^2+1}\mathrm{d}u$. *[3 marks]*

Markscheme

let

$u = x - \frac{1}{2}$ ***A1***

$\frac{du}{dx} = 1 \quad (\text{or } du = dx)$ ***A1***

$\int \frac{1}{4x^2-4x+5} dx = \int \frac{1}{4\left(x-\frac{1}{2}\right)^2+4} dx$ ***A1***

$\int \frac{1}{4u^2+4} du = \frac{1}{4} \int \frac{1}{u^2+1} du$ ***AG***

Note: If following through an incorrect answer to part (a), do not award final ***A1*** mark.

[3 marks]

12f. Prove that $\int_1^{3.5} \frac{1}{4x^2-4x+5} dx = \frac{\pi}{16}$. *[7 marks]*

Markscheme

$\int_1^{3.5} \frac{1}{4x^2-4x+5} dx = \frac{1}{4} \int_{0.5}^{3} \frac{1}{u^2+1} du$ ***A1***

Note: ***A1*** for correct change of limits. Award also if they do not change limits but go back to *x* values when substituting the limit (even if there is an error in the integral).

$\frac{1}{4}[\arctan(u)]_{0.5}^{3}$ ***(M1)***

$\frac{1}{4}\left(\arctan(3) - \arctan\left(\frac{1}{2}\right)\right)$ ***A1***

let the integral = *I*

$\tan 4I = \tan\left(\arctan(3) - \arctan\left(\frac{1}{2}\right)\right)$ ***M1***

$\frac{3-0.5}{1+3\times0.5} = \frac{2.5}{2.5} = 1$ ***(M1)A1***

$4I = \frac{\pi}{4} \Rightarrow I = \frac{\pi}{16}$ ***A1AG***

[7 marks]
