HL / Integration 2 *[131 marks]*

1. Find $\int \arcsin x \, dx$ *[5 marks]*

2a. Using the substitution $x = \tan\theta$ show that $\int_0^1 \frac{1}{(x^2+1)^2} dx = \int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$. *[4 marks]*

2b. Hence find the value of $\int_0^1 \frac{1}{(x^2+1)^2} dx$. *[3 marks]*

3a. Find $\int (1 + \tan^2 x) dx$. *[2 marks]*

3b. Find $\int \sin^2 x dx$. *[3 marks]*

4. By using the substitution $t = \tan x$, find $\int \frac{dx}{1+\sin^2 x}$. *[8 marks]*

Express your answer in the form $m \arctan(n \tan x) + c$, where m, n are constants to be determined.

5. Use the substitution $x = a \sec\theta$ to show that $\int_{a\sqrt{2}}^{2a} \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{1}{24a^3}\left(3\sqrt{3} + \pi - 6\right)$. *[7 marks]*

6. A particle moves in a straight line such that at time t seconds $(t \geqslant 0)$, its velocity v, in ms^{-1}, is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second. *[5 marks]*

7a. Express $x^2 + 3x + 2$ in the form $(x + h)^2 + k$. *[1 mark]*

7b. Factorize $x^2 + 3x + 2$. *[1 mark]*

Consider the function $f(x) = \frac{1}{x^2+3x+2}$, $x \in \mathbb{R}$, $x \neq -2$, $x \neq -1$.

7c. Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y-intercept and the local maximum. *[5 marks]*

7d. Show that $\frac{1}{x+1} - \frac{1}{x+2} = \frac{1}{x^2+3x+2}$. *[1 mark]*

7e. Hence find the value of p if $\int_0^1 f(x) dx = \ln(p)$. *[4 marks]*

7f. Sketch the graph of $y = f(|x|)$. *[2 marks]*

7g. Determine the area of the region enclosed between the graph of $y = f(|x|)$, the x-axis and the lines with equations $x = -1$ and $x = 1$. *[3 marks]*

A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t,\ t \geqslant 0$. The first two times when the particle is at rest are denoted by t_1 and t_2, where $t_1 < t_2$.

8a. Find t_1 and t_2. *[5 marks]*

8b. Find the displacement of the particle when $t = t_1$ *[2 marks]*

Consider the curve defined by the equation $4x^2 + y^2 = 7$.

9a. Find the equation of the normal to the curve at the point $\left(1,\ \sqrt{3}\right)$. *[6 marks]*

9b. Find the volume of the solid formed when the region bounded by the curve, the x-axis for $x \geqslant 0$ and the y-axis for $y \geqslant 0$ is rotated through 2π about the x-axis. *[3 marks]*

Let the function f be defined by $f(x) = \frac{2-e^x}{2e^x-1},\ x \in D$.

10a. Determine D, the largest possible domain of f. *[2 marks]*

10b. Show that the graph of f has three asymptotes and state their equations. *[5 marks]*

10c. Show that $f'(x) = -\frac{3e^x}{(2e^x-1)^2}$. *[3 marks]*

10d. Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. *[4 marks]*

10e. Find an expression for $f^{-1}(x)$. *[4 marks]*

10f. Consider the region R enclosed by the graph of $y = f(x)$ and the axes. *[4 marks]*

Find the volume of the solid obtained when R is rotated through 2π about the y-axis.

Consider the function defined by $f(x) = x\sqrt{1-x^2}$ on the domain $-1 \leq x \leq 1$.

11a. Show that f is an odd function. *[2 marks]*

11b. Find $f'(x)$. *[3 marks]*

11c. Hence find the x-coordinates of any local maximum or minimum points. *[3 marks]*

11d. Find the range of f. *[3 marks]*

11e. Sketch the graph of $y = f(x)$ indicating clearly the coordinates of the x-intercepts and any local maximum or minimum points. *[3 marks]*

11f. Find the area of the region enclosed by the graph of $y = f(x)$ and the x-axis for $x \geq 0$. *[4 marks]*

11g. Show that $\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\mathrm{d}x > \left|\int_{-1}^{1}x\sqrt{1-x^2}\mathrm{d}x\right|$. *[2 marks]*

12a. Express $4x^2 - 4x + 5$ in the form $a(x-h)^2 + k$ where *a*, *h*, $k \in \mathbb{Q}$. *[2 marks]*

12b. The graph of $y = x^2$ is transformed onto the graph of $y = 4x^2 - 4x + 5$. Describe a sequence of transformations that does this, making the order of transformations clear. *[3 marks]*

The function *f* is defined by $f(x) = \frac{1}{4x^2 - 4x + 5}$.

12c. Sketch the graph of $y = f(x)$. *[2 marks]*

12d. Find the range of *f*. *[2 marks]*

12e. By using a suitable substitution show that $\int f(x)\mathrm{d}x = \frac{1}{4}\int \frac{1}{u^2+1}\mathrm{d}u$. *[3 marks]*

12f. Prove that $\int_1^{3.5} \frac{1}{4x^2-4x+5}\mathrm{d}x = \frac{\pi}{16}$. *[7 marks]*