

HL / Integrals 2 [55 marks]

1a. Use the substitution $u = x^{\frac{1}{2}}$ to find $\int \frac{dx}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$. [4 marks]

1b. Hence find the value of $\frac{1}{2} \int_1^9 \frac{dx}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$, expressing your answer in the form $\arctan q$, where $q \in \mathbb{Q}$. [3 marks]

2. By using the substitution $x^2 = 2 \sec \theta$, show that $\int \frac{dx}{x\sqrt{x^2-4}} = \frac{1}{4} \arccos\left(\frac{2}{x^2}\right) + c$. [7 marks]

3. Using integration by parts find $\int x \sin x dx$. [4 marks]

4. Use the substitution $u = \ln x$ to find the value of $\int_{e^2}^e \frac{dx}{x \ln x}$. [4 marks]

5. Show that $\int_1^2 x^3 \ln x dx = 4 \ln 2 - \frac{15}{16}$. [6 marks]

6. By using the substitution $t = \tan x$, find $\int \frac{dx}{1 + \sin^2 x}$. [8 marks]
Express your answer in the form $m \arctan(n \tan x) + c$, where m, n are constants to be determined.

7a. Express $4x^2 - 4x + 5$ in the form $a(x - h)^2 + k$ where $a, h, k \in \mathbb{Q}$. [2 marks]

7b. The graph of $y = x^2$ is transformed onto the graph of $y = 4x^2 - 4x + 5$. Describe a sequence of transformations that does this, making the order of transformations clear. [3 marks]

The function f is defined by

$$f(x) = \frac{1}{4x^2 - 4x + 5}.$$

7c. Sketch the graph of $y = f(x)$. [2 marks]

7d. Find the range of f . [2 marks]

7e. By using a suitable substitution show that $\int f(x) dx = \frac{1}{4} \int \frac{1}{u^2 + 1} du$. [3 marks]

7f. Prove that $\int_1^{3.5} \frac{1}{4x^2 - 4x + 5} dx = \frac{\pi}{16}$. [7 marks]

