# Differential Calculus - Test 2

Syllabus content on test:

- Finding equations of a tangents and normal
- Chain rule for composite functions
- Product and quotient rules
- Maximum and minimum points; testing for maximum and minimum
- 2nd derivative and points of inflexion
- Optimization

**Part I** - No calculator allowed for questions 1 – 4.

**1.** Find the derivative of each of the following functions.

(a) $y = x^2 \ln x$

(b) $f(x) = \dfrac{x^3 - 2x + 5}{x}$

(c) $g(x) = e^x \sin 2x$

(d) $y = \dfrac{10 - 4x}{e^{x/2}}$

**2.** If $f(x) = \cos(2x)$ and line $L$ is tangent to $f$ at the point where $x = \dfrac{\pi}{4}$. Find the equation of $L$.

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**3.** Consider the function $g(x)=\frac{e^{3x}}{x}$.

(a) Find $g'(x)$.

(b) The graph of $g$ has a minimum point at $A$. Find the coordinates of $A$.

(c) Find the equation of the line that is normal to the graph of $g$ at the point where $x=\frac{1}{3}$.

**4.** Consider the function $f(x)=x\ln(x^2)$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Does the graph of $f$ have an inflexion point. Explain.

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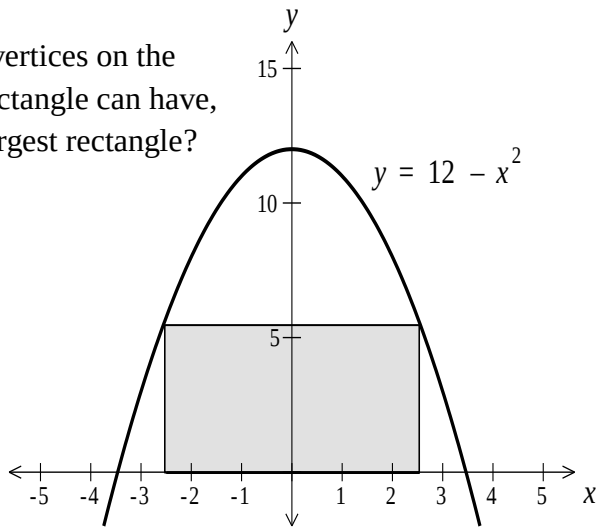
**Part II** - A calculator is allowed for questions 5 – 8.

**5.** A rectangle has its base on the *x*-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area that the rectangle can have, and what are the dimensions (length and width) of the largest rectangle?




**6.** The graph of the equation $y = e^x\left(1 - x^2\right)$ has two points of inflexion. Find the coordinates of each - approximating the *x*- and *y*-coordinates to three significant figures.

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**7**. A rectangular box has height $h$ cm, width $x$ cm and length $2x$ cm. It is designed to have a volume equal to 1 litre (1000 $\text{cm}^3$).

(a) Show that $h = \dfrac{500}{x^2}$ cm.

(b) Find an expression for the total surface area, $S$ $\text{cm}^2$, of the box in terms of $x$.

(c) Find the dimensions of the box that produces a minimum surface area.

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**8.** A curve has equation $y = -x(x+5)^2$.

(a) For this curve find:

(i) the $x$-intercepts;

(ii) the exact coordinates of the maximum point;

(iii) the exact coordinates of the point of inflection.

**Bonus**

Find the equation of the line that is tangent to the graph of $y = x\tan x$ at the point where $x = \frac{\pi}{4}$.