

## Differential Calculus - Test 1

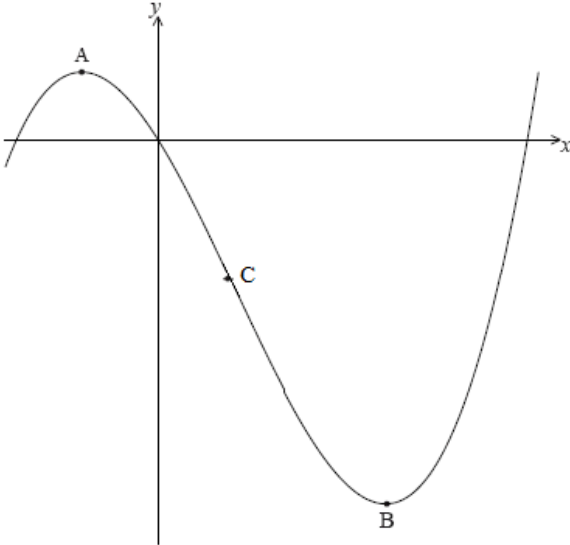
Syllabus content on test:

- Finding derivative from first principles (limit definition of derivative)
- Derivative of  $x^n$  ▪ Derivative interpreted as gradient function ▪ Chain rule for composite functions
- Stationary points; maximum & minimum points; testing for max. and min. ▪ 2<sup>nd</sup> derivative and points of inflexion ▪ Finding equation of tangent line

### Part I - No calculator allowed for questions 1 – 4.

**total marks on test: 55**

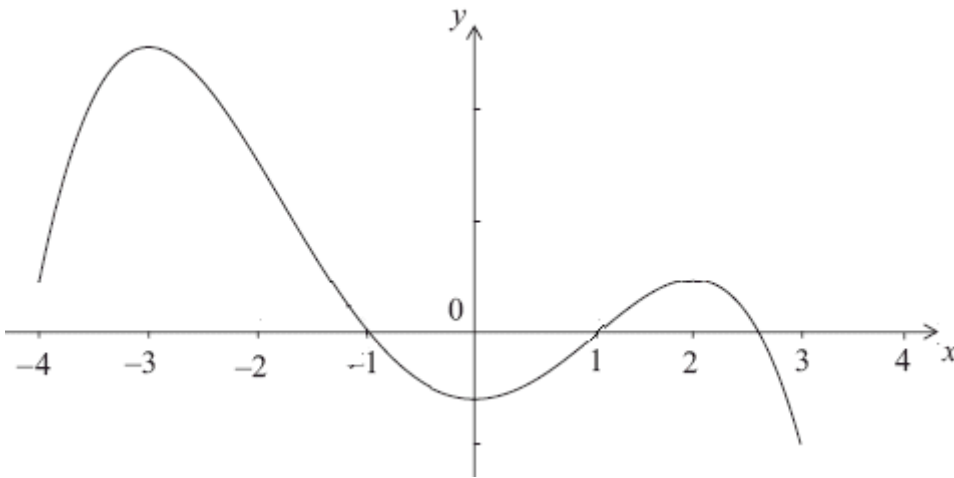
1. Let  $f(x) = \frac{1}{3}x^3 - x^2 - 3x$ . The graph of  $f$  has a maximum at point A, a minimum at point B, and an inflexion point at point C. Find the coordinates of the three points A, B and C. [ 6 marks ]



2. Consider the function  $g(x) = 8x^2 + \frac{c}{x}$ ,  $x \neq 0$ , where  $c$  is a constant. [ 7 marks ]
- (a) Find  $g'(x)$
- (b) There is a minimum value of  $g(x)$  when  $x = \frac{1}{2}$ . Find the value of  $c$ .

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3. A function  $h(x)$  is defined for  $-4 \leq x \leq 3$ . The graph of  $h(x)$  is given below. [ 8 marks ]



The graph of  $h(x)$  has a local minimum at  $x = 0$ , and local maxima when  $x = -3$  and  $x = 2$

- Write down the  $x$ -intercepts of the graph of the **derivative** function,  $h'(x)$ .
- Write down all values of  $x$  for which  $h'(x)$  is positive.
- Explain why  $h''(2.5) < 0$ .

4. Consider the function  $f(x) = x^2 - 5x + 2$ . Use the formula  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to show that the derivative of  $f(x)$  is  $2x - 5$ .

[ 6 marks ]

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### Part II – Calculator allowed for questions 5-8

5. Consider the cubic function  $f(x) = 2x^3 + 3x^2 - 12x + 4$ . [ 8 marks ]
- (a) Find the coordinates of any stationary points and confirm whether they are a maximum, minimum or neither.
- (b) Find the coordinates of any inflexion point. Justify that the concavity of  $f$  changes at any inflexion point you find.

6. Find the equation of the line tangent to  $y = \sqrt{2-4x}$  at the point  $\left(\frac{1}{4}, 1\right)$ . [ 8 marks ]

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7. Explain why the graph of the function  $g(x) = \frac{1}{3x+6}$  has no stationary points. [ 5 marks ]

8. The curve  $y = x^3 + ax^2 + 2x - 1$  has an inflexion point at  $x = -3$ . Find the value of  $a$ . [7 marks]