

Differential Calculus – Test 2

Part I - Questions 1-4: NO calculator

7 questions >> 65 total marks

- Given that $f(x) = x^2 - 3x$, find $f'(x)$ from first principles; that is, use the limit definition of the derivative $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. [8 marks]
- Find the equation of the line tangent to the curve $y = \frac{x-4}{x+2}$ at the point where $x = 3$. [8 marks]
- Find the equation of the line normal to $y = \sqrt{1-4x}$ at the point $(-2, 3)$. [8 marks]
- The line L with equation $y = x - 5$ intersects the graph of $g(x) = 2x^3 + ax^2 + bx + 3$ at $(-1, -6)$. Line L is also tangent to the graph of g at $x = 2$. Find the value of a and the value of b . [8 marks]

Part II - Questions 5-8: Calculator allowed

- Consider the function $y = e^x \cos x$ with domain $0 < x < 5$. [15 marks]
 - Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
 - Find the approximate coordinates of any maximum or minimum point for the function in the indicated domain. Provide a brief justification for each.
 - Find the exact coordinates of any inflexion point for the function in the indicated domain. Provide a brief justification for each.
- A rectangle is circumscribed by the graph of the parabola $y = 3x - \frac{x^2}{2}$ such that the two upper vertices of the rectangle are on the graph of the parabola and the two lower vertices are on the x -axis (as shown in the diagram). The lower left vertex of the rectangle has coordinates $(c, 0)$.
 - Show that the function, A , for the area of the rectangle expressed in terms of c is $A(c) = c^3 - 9c^2 + 18c$.
 - Find the width and height of the rectangle with the maximum area.
 - Find the maximum area.
- Find the coordinates of the two points on the graph of $y = 9 - x^2$ that are closest to $(0, 4)$. [8 marks]

