■ sample question #3 ■

[Maximum mark: 28]

This question asks you to investigate a two-player dice game. During a player's turn, **two** fair six-sided dice are rolled and the sum of the two numbers showing are added to the player's score for that turn. If either of the dice show a six, then the player's turn is ended with a score of zero for that turn and the dice are passed to the other player. A player may continue rolling the pair of dice as many times as desired during a turn. If a player chooses to finish the turn before rolling a six, the score for that turn is added to the sum of scores from previous turns to give a total score. The first player to have a total score of 100 or more wins the game.

Here is an example of what the beginning of a game may look like.

turn	#s on dice	decision	score for turn	total score player A	total score player B
1. player A	3,5	continue rolling		13	
	4,1	stop rolling	13		
1. player B	2,2	continue rolling			0
	4,6	(6 ends turn)	0		
2. player A	6,3	(6 ends turn)	0	13	
2. player B	4,4	continue rolling			21
	5,1	continue rolling			
	3,4	stop rolling	21		
3. player A	4,5	stop rolling	9	22	
3. player B	1,1	continue rolling			21
	3,5	continue rolling			
	4,1	continue rolling			
	6,6	(6 ends turn)	0		

(a) The ten possible scores for one roll of the two dice are zero and the integers from 2 to 10 inclusive. Let X be a discrete random variable that represents the score on one roll of the two dice and $\mathrm{P}(X = x)$ is the probability that X is equal to one of the values of the random variable.

Copy and complete the table below that gives the probability distribution for X. [4]

x	0	2	3	4	5	6	7	8	9	10
$\mathrm{P}(X = x)$		$\frac{1}{36}$								

[question continued on next page]

(b) Find the expected score after the first roll of the two dice during a turn. [4]

(c) Find the probability of no sixes showing for one roll of the two dice. [2]

(d) Given that no sixes occur for a given roll of the two dice, the possible scores are from 2 to 10 inclusive. Let Y be a discrete random variable that represents the score on one roll of the two dice **given than no sixes occur**. Copy and complete the table below that gives the probability distribution for Y. [4]

y	2	3	4	5	6	7	8	9	10
$\mathrm{P}(Y = y)$	$1/25$								

(e) (i) Show that the expected score for one roll of the two dice, given that no sixes have occurred, is 6. [2]

(ii) Explain why, after n rolls with no sixes, the expected total score for the turn is $6n$. [1]

(f) Let μ_n be the expected score after n rolls in a turn that does **not** assume that no sixes have occurred. Find an expression for μ_n in terms of n. [3]

(g) Explain why the best strategy is to attempt to make three rolls each turn – and to stop your turn once three rolls of the two dice have been made. [3]

(h) Consider the same game but played by rolling just one die rather than two dice. Determine the best strategy for this game. [5]