Mathematics: analysis and approaches
Higher level
Paper 3

v3

■ sample question #1 ■

[Maximum mark: 30]

Consider the curve $y = xe^x$ and the line $y = kx,\ k \in \mathbb{R}$.

(a) Let $k = 0$. Show that the curve and the line intersect only once. [2]

(b) Let $k = 1$. Show that the line is a tangent to the curve. [2]

(c) (i) Find the values of *k* such that the curve $y = xe^x$ and the line $y = kx$ intersect at two distinct points.

(ii) Write down the coordinates of the two points of intersection. [5]

(d) Let A be the region enclosed by the curve and the line when $k > 1$.

(i) Write down an integral representing the area of A.

(ii) Find the **exact** area of A when $k = e^2$.

(iii) Show that the area of A when $k = e^n,\ n \in \mathbb{R}^+$, is equal to $e^n\left(\frac{n^2}{2} - n + 1\right) - 1$. [9]

(e) The curve has a horizontal tangent at the point P.

(i) Find the **exact** value of *k* such that the line $y = kx$ passes through P.

(ii) For the value of *k* found in part (e)(i), show that the area of the region enclosed by the curve and the line is $1 - \frac{5}{2e}$. [7]

(e) Let B be the region enclosed by the curve and the line when $0 < k < 1$. Show that the area of B must be less than 1. [5]