

■ sample question #5 ■

5. [Maximum mark: 30]

Consider the function defined as $h(x) = \frac{2}{3 + e^{-2x}}$.

(a) State the domain and range of h . [2]

(b) Find $h'(x)$, the first derivative of h . [3]

(c) Show that the second derivative of h is $h''(x) = \frac{-8e^{-2x}(3 - e^{-2x})}{(3 + e^{-2x})^3}$. [3]

(d) Point A is the point on the graph of h where $h''(x) = 0$. Find the exact coordinates of A. [3]

(e) Show that A is a point of inflexion. [3]

(f) The line L is tangent to the graph of h at A. Find the equation of L and express it in the form $y = mx + c$. [4]

Consider the function defined as $h(x) = \frac{a}{b + e^{-cx}}$, where $a \neq 0$, $b > 0$, $c > 0$.

(g) State the domain and range of h . [2]

(h) Show that $h''(x) = \frac{ac^2e^{-cx}(e^{-cx} - b)}{(b + e^{-cx})^3}$. [4]

(i) Find the coordinates of the point on the graph of h where $h''(x) = 0$. [3]

(j) Show that this is a point of inflexion. [3]