HL / paper 1 / 23.11.2022 *[96 marks]*

1. Use the principle of mathematical induction to prove that *[7 marks]*

$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+\ldots+n\left(\frac{1}{2}\right)^{n-1}=4-\frac{n+2}{2^{n-1}}$, where $n\in\mathbb{Z}^+$.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

if $n=1$

$\text{LHS}=1\,;\,\text{RHS}=4-\frac{3}{2^0}=4-3=1$ ***M1***

hence true for $n=1$

assume true for $n=k$ ***M1***

Note: Assumption of truth must be present. Following marks are not dependent on the first two ***M1*** marks.

so $1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+\ldots+k\left(\frac{1}{2}\right)^{k-1}=4-\frac{k+2}{2^{k-1}}$

if $n=k+1$

$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+\ldots+k\left(\frac{1}{2}\right)^{k-1}+(k+1)\left(\frac{1}{2}\right)^k$

$=4-\frac{k+2}{2^{k-1}}+(k+1)\left(\frac{1}{2}\right)^k$ ***M1A1***

finding a common denominator for the two fractions ***M1***

$=4-\frac{2(k+2)}{2^k}+\frac{k+1}{2^k}$

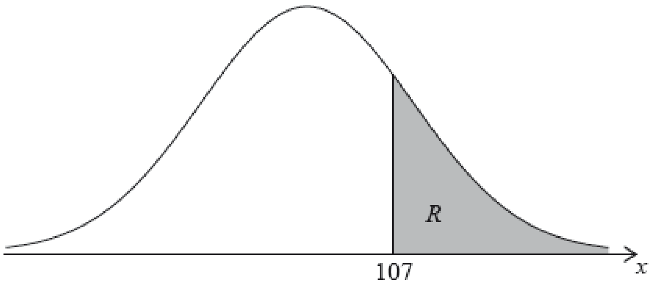
$=4-\frac{2(k+2)-(k+1)}{2^k}=4-\frac{k+3}{2^k}\left(=4-\frac{(k+1)+2}{2^{(k+1)-1}}\right)$ ***A1***

hence if true for $n=k$ then also true for $n=k+1$, as true for $n=1$, so true (for all $n\in\mathbb{Z}^+$) ***R1***

Note: Award the final ***R1*** only if the first four marks have been awarded.

[7 marks]

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

2a. Write down $\mathrm{P}(X > 107)$. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\mathrm{P}(X > 107) = 0.24 \quad \left(= \frac{6}{25},\ 24\%\right)$ ***A1 N1***

[1 mark]

2b. Find $\mathrm{P}(100 < X < 107)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(X > 100) = 0.5,\ \mathrm{P}(X > 100) - \mathrm{P}(X > 107)$

correct working ***(A1)***

eg $0.5 - 0.24,\ 0.76 - 0.5$

$\mathrm{P}(100 < X < 107) = 0.26 \quad \left(= \frac{13}{50},\ 26\%\right)$ ***A1 N2***

[3 marks]

2c. Find $\mathrm{P}(93 < X < 107)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $2 \times 0.26,\ 1 - 2(0.24),\ \mathrm{P}(93 < X < 100) = \mathrm{P}(100 < X < 107)$

$\mathrm{P}(93 < X < 107) = 0.52 \quad \left(= \frac{13}{25},\ 52\%\right)$ ***A1 N2***

[2 marks]

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin\left(\frac{\pi x}{6}\right), & 0 \leqslant x \leqslant 6 \\ 0, & \text{otherwise} \end{cases}.$$

3a. Find the value of k. *[4 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to equate integral to 1 (may appear later) ***M1***

$k\int_0^6 \sin\left(\frac{\pi x}{6}\right)\mathrm{d}x = 1$

correct integral ***A1***

$k\left[-\frac{6}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^6 = 1$

substituting limits ***M1***

$-\frac{6}{\pi}(-1 - 1) = \frac{1}{k}$

$k = \frac{\pi}{12}$ ***A1***

[4 marks]

3b. Show that $P(0 \leqslant X \leqslant 2) = \frac{1}{4}$. *[4 marks]*

Markscheme

$\frac{\pi}{12}\int_0^2 \sin\left(\frac{\pi x}{6}\right) \mathrm{d}x$ ***M1***

$= \frac{\pi}{12}\left[-\frac{6}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^2$ ***A1***

Note: Accept without the $\frac{\pi}{12}$ at this stage if it is added later.

$\frac{\pi}{12}\left[-\frac{6}{\pi}\left(\cos\frac{\pi}{3} - 1\right)\right]$ ***M1***

$= \frac{1}{4}$ ***AG***

[4 marks]

3c. Hence state the interquartile range of X. *[2 marks]*

Markscheme

from (c)(i) $Q_1 = 2$ ***(A1)***

as the graph is symmetrical about the middle value $x = 3 \Rightarrow Q_3 = 4$ ***(A1)***

so interquartile range is

$4 - 2$

$= 2$ ***A1***

[3 marks]

3d. Calculate $P(X \leqslant 4 | X \geqslant 3)$. *[2 marks]*

Markscheme

$P(X \leqslant 4 | X \geqslant 3) = \frac{P(3 \leqslant X \leqslant 4)}{P(X \geqslant 3)}$

$= \frac{\frac{1}{4}}{\frac{1}{2}}$ ***(M1)***

$= \frac{1}{2}$ ***A1***

[2 marks]

4. Use the method of mathematical induction to prove that $4^n + 15n - 1$ is divisible by 9 for $n \in \mathbb{Z}^+$. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

let $P(n)$ be the proposition that $4^n + 15n - 1$ is divisible by 9

showing true for $n = 1$ ***A1***

*ie*for $n = 1,\ 4^1 + 15 \times 1 - 1 = 18$

which is divisible by 9, therefore $P(1)$ is true

assume $P(k)$ is true so $4^k + 15k - 1 = 9A,\ (A \in \mathbb{Z}^+)$ ***M1***

Note: Only award ***M1*** if "truth assumed" or equivalent.

consider $4^{k+1} + 15(k+1) - 1$

$= 4 \times 4^k + 15k + 14$

$= 4(9A - 15k + 1) + 15k + 14$ ***M1***

$= 4 \times 9A - 45k + 18$ ***A1***

$= 9(4A - 5k + 2)$ which is divisible by 9 ***R1***

Note: Award ***R1*** for either the expression or the statement above.

since $P(1)$ is true and $P(k)$ true implies $P(k+1)$ is true, therefore (by the principle of mathematical induction) $P(n)$ is true for $n \in \mathbb{Z}^+$ ***R1***

Note: Only award the final ***R1*** if the 2 ***M1***s have been awarded.

[6 marks]

Consider the three planes

$\prod_1 :\ 2x - y + z = 4$

$\prod_2 :\ x - 2y + 3z = 5$

$\prod_3 : -9x + 3y - 2z = 32$

5a. Show that the three planes do not intersect. *[4 marks]*

Markscheme

METHOD 1

attempt to eliminate a variable ***M1***

obtain a pair of equations in two variables

EITHER

$-3x + z = -3$ and ***A1***

$-3x + z = 44$ ***A1***

OR

$-5x + y = -7$ and ***A1***

$-5x + y = 40$ ***A1***

OR

$3x - z = 3$ and ***A1***

$3x - z = -\frac{79}{5}$ ***A1***

THEN

the two lines are parallel ($-3 \neq 44$ or $-7 \neq 40$ or $3 \neq -\frac{79}{5}$) ***R1***

Note: There are other possible pairs of equations in two variables.
To obtain the final ***R1***, at least the initial ***M1*** must have been awarded.

hence the three planes do not intersect ***AG***

METHOD 2

vector product of the two normals $= \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix}$ (or equivalent) ***A1***

$r = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$ (or equivalent) ***A1***

Note: Award ***A0*** if "$r =$" is missing. Subsequent marks may still be awarded.

Attempt to substitute $(1+\lambda, -2+5\lambda, 3\lambda)$ in $\prod_3$ ***M1***

$-9(1+\lambda)+3(-2+5\lambda)-2(3\lambda)=32$

$-15=32$, a contradiction ***R1***

hence the three planes do not intersect ***AG***

METHOD 3

attempt to eliminate a variable ***M1***

$-3y+5z=6$ ***A1***

$-3y+5z=100$ ***A1***

$0=94$, a contradiction ***R1***

Note: Accept other equivalent alternatives. Accept other valid methods.
To obtain the final ***R1***, at least the initial ***M1*** must have been awarded.

hence the three planes do not intersect ***AG***

[4 marks]

5b. Verify that the point $\mathrm{P}(1,\ -2,\ 0)$ lies on both $\prod_1$ and $\prod_2$. *[1 mark]*

Markscheme

$\prod_1 : 2+2+0=4$ and $\prod_2 : 1+4+0=5$ ***A1***

[1 mark]

5c. Find a vector equation of L, the line of intersection of $\prod_1$ and $\prod_2$. *[4 marks]*

Markscheme

METHOD 1

attempt to find the vector product of the two normals ***M1***

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix}$$ ***A1***

$$r = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$ ***A1A1***

Note: Award ***A1A0*** if "$r =$" is missing.
Accept any multiple of the direction vector.
Working for (b)(ii) may be seen in part (a) Method 2. In this case penalize lack of "$r =$" only once.

METHOD 2

attempt to eliminate a variable from $\prod_1$ and $\prod_2$ ***M1***

$3x - z = 3$ OR $3y - 5z = -6$ OR $5x - y = 7$

Let $x = t$

substituting $x = t$ in $3x - z = 3$ to obtain

$z = -3 + 3t$ and $y = 5t - 7$ (for all three variables in parametric form) ***A1***

$$r = \begin{pmatrix} 0 \\ -7 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$ ***A1A1***

Note: Award ***A1A0*** if "$r =$" is missing.
Accept any multiple of the direction vector. Accept other position vectors which satisfy both the planes $\prod_1$ and $\prod_2$.

[4 marks]

Points A and B have coordinates $(1, 1, 2)$ and $(9, m, -6)$ respectively.

6a. Express $\overrightarrow{\mathrm{AB}}$ in terms of m. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach to find $\overrightarrow{\mathrm{AB}}$ ***(M1)***

eg $\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$, $\mathrm{A} - \mathrm{B}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 8 \\ m-1 \\ -8 \end{pmatrix}$ ***A1 N2***

[2 marks]

The line L, which passes through B, has equation $r = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$.

6b. Find the value of m. *[5 marks]*

Markscheme

valid approach ***(M1)***

eg $L = \begin{pmatrix} 9 \\ m \\ -6 \end{pmatrix}, \begin{pmatrix} 9 \\ m \\ -6 \end{pmatrix} = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$

one correct equation ***(A1)***

eg $-3 + 2s = 9, \; -6 = 24 - 5s$

correct value for s ***A1***

eg $s = 6$

substituting **their** s value into their expression/equation to find m ***(M1)***

eg $-19 + 6 \times 4$

$m = 5$ ***A1*** ***N3***

[5 marks]

6c. Consider a unit vector u, such that $u = pi - \frac{2}{3}j + \frac{1}{3}k$, where $p > 0$. *[8 marks]*

Point C is such that $\overrightarrow{\mathrm{BC}} = 9u$.

Find the coordinates of C.

Markscheme

valid approach ***(M1)***

eg $\overrightarrow{\mathrm{BC}}=\begin{pmatrix}9p\\-6\\3\end{pmatrix}$, $C=9u+B$, $\overrightarrow{\mathrm{BC}}=\begin{pmatrix}x-9\\y-5\\z+6\end{pmatrix}$

correct working to find C ***(A1)***

eg $\overrightarrow{\mathrm{OC}}=\begin{pmatrix}9p+9\\-1\\-3\end{pmatrix}$, $\mathrm{C}=9\begin{pmatrix}p\\-\frac{2}{3}\\\frac{1}{3}\end{pmatrix}+\begin{pmatrix}9\\5\\-6\end{pmatrix}$, $y=-1$ and $z=-3$

correct approach to find $|u|$ (seen anywhere) ***A1***

eg $p^2+\left(-\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2$, $\sqrt{p^2+\frac{4}{9}+\frac{1}{9}}$

recognizing unit vector has magnitude of 1 ***(M1)***

eg $|u|=1$, $\sqrt{p^2+\left(-\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2}=1$, $p^2+\frac{5}{9}=1$

correct working ***(A1)***

eg $p^2=\frac{4}{9}$, $p=\pm\frac{2}{3}$

$p=\frac{2}{3}$ ***A1***

substituting **their** value of p ***(M1)***

eg

$\begin{pmatrix}x-9\\y-5\\z+6\end{pmatrix}=\begin{pmatrix}6\\-6\\3\end{pmatrix}$, $\mathrm{C}=\begin{pmatrix}6\\-6\\3\end{pmatrix}+\begin{pmatrix}9\\5\\-6\end{pmatrix}$, $\mathrm{C}=9\begin{pmatrix}\frac{2}{3}\\-\frac{2}{3}\\\frac{1}{3}\end{pmatrix}+\begin{pmatrix}9\\5\\-6\end{pmatrix}$, $x-9=6$

$\mathrm{C}(15,\ -1,\ -3)$ (accept $\begin{pmatrix}15\\-1\\-3\end{pmatrix}$) ***A1*** ***N4***

Note: The marks for finding p are independent of the first two marks.
For example, it is possible to award marks such as ***(M0)(A0)A1(M1)(A1)A1 (M0)A0*** or ***(M0)(A0)A1(M1)(A0)A0 (M1)A0***.

[8 marks]

The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

Find, in terms of k,

7a. $\overrightarrow{\mathrm{OA}} \bullet \overrightarrow{\mathrm{OC}}$. *[2 marks]*

Markscheme

correct substitution into either $\overrightarrow{\mathrm{OA}} \bullet \overrightarrow{\mathrm{OC}}$ or into $\overrightarrow{\mathrm{OB}} \bullet \overrightarrow{\mathrm{OC}}$ (in (ii)) ***(A1)***

eg $-2 \times (-1) + 4 \times k,\ 6 \times (-1) + 8 \times k$

correct expression ***A1 N1***

eg $2 + 4k,\ 4k + 2$

[2 marks]

7b. $\overrightarrow{\mathrm{OB}} \bullet \overrightarrow{\mathrm{OC}}$. *[1 mark]*

Markscheme

correct expression ***A1 N1***

eg $8k - 6,\ -6 + 8k$

[1 mark]

7c. Given that $\mathrm{A}\hat{\mathrm{O}}\mathrm{C} = \mathrm{B}\hat{\mathrm{O}}\mathrm{C}$, show that $k = 7$. *[8 marks]*

Markscheme

finding magnitudes (seen anywhere) ***A1A1***

eg $\sqrt{(-2)^2+(4)^2+(-4)^2}$ $(=6)$, $\sqrt{(6)^2+(8)^2+0^2}$ $(=10)$

correct substitution of their values into formula for angle AOC ***(A1)***

eg $\cos\theta = \frac{2+4k}{\sqrt{(-2)^2+(4)^2+(-4)^2}\left|\overrightarrow{\mathrm{OC}}\right|}$

correct substitution of their values into formula for angle BOC ***(A1)***

eg $\cos\theta = \frac{8k-6}{\sqrt{(6)^2+(8)^2+0^2}\left|\overrightarrow{\mathrm{OC}}\right|}$

recognizing that $\cos\mathrm{A\hat{O}C} = \cos\mathrm{B\hat{O}C}$ (seen anywhere) ***(M1)***

eg $\frac{2+4k}{\left|\overrightarrow{\mathrm{OC}}\right|\sqrt{(-2)^2+(4)^2+(-4)^2}} = \frac{8k-6}{\left|\overrightarrow{\mathrm{OC}}\right|\sqrt{6^2+(8)^2+0^2}}$, $\frac{2+4k}{6\sqrt{1+k^2}} = \frac{8k-6}{10\sqrt{1+k^2}}$

correct working (without radicals) ***(A2)***

eg $10(2+4k) = 6(8k-6)$, $11k^2-79k+14=0$

correct working clearly leading to the required answer ***A1***

eg $20+36=48k-40k$, $56=8k$, $k=7$ and $k=\frac{2}{11}$,
$(k-7)(11k-2)=0$

$k=7$ ***AG N0***

[8 marks]

7d. Calculate the area of triangle AOC. *[6 marks]*

Markscheme

finding magnitude of $\overrightarrow{\mathrm{OC}}$ (seen anywhere) ***A1***

eg $\sqrt{(-1)^2+7^2+0^2},\ \sqrt{50}$

valid attempt to find $\cos\theta$ ***(M1)***

eg $\cos\theta=\frac{2+28}{6\sqrt{(-1)^2+7^2+0^2}},\ \cos\theta=\frac{56-6}{10\sqrt{(-1)^2+7^2+0^2}},$

$\left(\sqrt{26}\right)^2=6^2+\left(\sqrt{50}\right)^2-2(6)\sqrt{50}\cos\theta$

finding $\cos\theta$ ***A1***

eg $\cos\theta=\frac{5}{\sqrt{50}}\ \left(=\frac{1}{\sqrt{2}}\right)$

valid approach to find $\sin\theta$ (seen anywhere) ***(M1)***

eg $\theta=\frac{\pi}{4},\ \sin\theta=\cos\theta,\ \sin\theta=\sqrt{1-\frac{25}{50}},\ \sin\theta=\sqrt{1-\cos^2\theta},$ $\sin\theta=\frac{\sqrt{2}}{2}$

correct substitution of **their** values into $\frac{1}{2}ab\sin\mathrm{C}$ ***(A1)***

eg $\frac{1}{2}\times6\times\sqrt{50}\times\sqrt{1-\frac{25}{50}},\ \frac{1}{2}\times6\times\sqrt{50}\times\frac{5}{\sqrt{50}}$

area is 15 ***A1 N3***

[6 marks]

A line, L_1, has equation $r=\begin{pmatrix}-3\\9\\10\end{pmatrix}+s\begin{pmatrix}6\\0\\2\end{pmatrix}$. Point $\mathrm{P}\,(15,\,9,\,c)$ lies on L_1.

8a. Find c. *[4 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct equation ***(A1)***

eg $-3+6s=15,\ 6s=18$

$s=3$ ***(A1)***

substitute their s value into z component ***(M1)***

eg $10+3(2),\ 10+6$

$c=16$ ***A1 N3***

[4 marks]

8b. A second line, L_2, is parallel to L_1 and passes through (1, 2, 3). *[2 marks]*

Write down a vector equation for L_2.

Markscheme

$r=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$ (=(***i*** + 2***j*** + 3***k***) + t(6***i*** + 2***k***)) ***A2 N2***

Note: Accept any scalar multiple of $\begin{pmatrix}6\\0\\2\end{pmatrix}$ for the direction vector.

Award **A1** for $\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, ***A1*** for $L_2=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, ***A0*** for $r=\begin{pmatrix}6\\0\\2\end{pmatrix}+t\begin{pmatrix}1\\2\\3\end{pmatrix}$.

[2 marks]

Three points in three-dimensional space have coordinates A(0, 0, 2), B(0, 2, 0) and C(3, 1, 0).

9a. Find the vector $\overrightarrow{AB}$. *[1 mark]*

Markscheme

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$ ***A1***

Note: Accept row vectors or equivalent.

[1 mark]

9b. Find the vector $\overrightarrow{AC}$. *[1 mark]*

Markscheme

$\overrightarrow{AC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ ***A1***

Note: Accept row vectors or equivalent.

[1 mark]

9c. Hence or otherwise, find the area of the triangle ABC. *[4 marks]*

Markscheme

METHOD 1

attempt at vector product using $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. ***(M1)***

$\pm(2\boldsymbol{i} + 6\boldsymbol{j} + 6\boldsymbol{k})$ ***A1***

attempt to use area $= \frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|$ ***M1***

$= \frac{\sqrt{76}}{2}\ \left(= \sqrt{19}\right)$ ***A1***

METHOD 2

attempt to use $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = \left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right|\cos\theta$ ***M1***

$$\begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \sqrt{0^2 + 2^2 + (-2)^2}\sqrt{3^2 + 1^2 + (-2)^2}\cos\theta$$

$6 = \sqrt{8}\sqrt{14}\cos\theta$ ***A1***

$\cos\theta = \frac{6}{\sqrt{8}\sqrt{14}} = \frac{6}{\sqrt{112}}$

attempt to use area $= \frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|\sin\theta$ ***M1***

$$= \frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{1 - \frac{36}{112}}\ \left(= \frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{\frac{76}{112}}\right)$$

$= \frac{\sqrt{76}}{2}\ \left(= \sqrt{19}\right)$ ***A1***

[4 marks]

The vectors $\boldsymbol{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other.

10a. Find the value of k. *[4 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of scalar product ***M1***

eg $\boldsymbol{a} \bullet \boldsymbol{b}$, $4(k+3)+2k$

recognizing scalar product must be zero ***(M1)***

eg $\boldsymbol{a} \bullet \boldsymbol{b} = 0, 4k+12+2k=0$

correct working (must involve combining terms) ***(A1)***

eg $6k+12,\ \ 6k=-12$

$k=-2$ ***A1 N2***

[4 marks]

10b. Given that $\boldsymbol{c} = \boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{c}$. *[3 marks]*

Markscheme

attempt to substitute **their** value of k (seen anywhere) ***(M1)***

eg $\boldsymbol{b} = \begin{pmatrix} -2+3 \\ -2 \end{pmatrix}$, $2\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

correct working ***(A1)***

eg $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 4+2k+6 \\ 2+2k \end{pmatrix}$

$\boldsymbol{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ ***A1 N2***

[3 marks]

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 2\boldsymbol{k}$.

11a. Find $\boldsymbol{a} \times \boldsymbol{b}$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\boldsymbol{a} \times \boldsymbol{b} = -12\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ ***(M1)A1***

[2 marks]

11b. Hence find the Cartesian equation of the plane containing the vectors ***a*** and ***b***, and passing through the point $(1, 0, -1)$. *[3 marks]*

Markscheme

METHOD 1

$-12x - 2y - 3z = d$ ***M1***

$-12 \times 1 - 2 \times 0 - 3(-1) = d$ ***(M1)***

$\Rightarrow d = -9$ ***A1***

$-12x - 2y - 3z = -9(\text{or} 12x + 2y + 3z = 9)$

METHOD 2

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} -12 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -12 \\ -2 \\ -3 \end{pmatrix}$ ***M1A1***

$-12x - 2y - 3z = -9(\text{or} 12x + 2y + 3z = 9)$ ***A1***

[3 marks]