HL / Revision / 21.11.2022 *[36 marks]*

1. Consider the function $f\left(x\right) = x\,\mathrm{e}^{2x}$, where $x \in \mathbb{R}$. The n^{th} derivative of $f\left(x\right)$ is denoted by $f^{(n)}\left(x\right)$. *[7 marks]*

Prove, by mathematical induction, that $f^{(n)}\left(x\right) = \left(2^n x + n2^{n-1}\right)\mathrm{e}^{2x}$, $n \in \mathbb{Z}^+$.

The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

2a. Find the probability that a male runner selected at random will complete the marathon in less than 3 hours. *[2 marks]*

It is found that 5% of the male runners complete the marathon in less than T_1 minutes.

2b. Calculate T_1. *[2 marks]*

The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

2c. Find the standard deviation of the times taken by female runners. *[4 marks]*

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin\left(\frac{\pi x}{6}\right), & 0 \leqslant x \leqslant 6 \\ 0, & \text{otherwise} \end{cases}.$$

3a. Find the value of k. *[4 marks]*

3b. By considering the graph of f write down the mean of X; *[1 mark]*

3c. By considering the graph of *f* write down the median of X; *[1 mark]*

3d. By considering the graph of *f* write down the mode of X. *[1 mark]*

3e. Show that $P(0 \leqslant X \leqslant 2) = \frac{1}{4}$. *[4 marks]*

3f. Hence state the interquartile range of X. *[2 marks]*

3g. Calculate $P(X \leqslant 4 | X \geqslant 3)$. *[2 marks]*

The points A and B are given by $\mathrm{A}(0, 3, -6)$ and $\mathrm{B}(6, -5, 11)$.

The plane *Π* is defined by the equation $4x - 3y + 2z = 20$.

4a. Find a vector equation of the line *L* passing through the points A and B. *[3 marks]*

4b. Find the coordinates of the point of intersection of the line *L* with the plane *Π*. *[3 marks]*