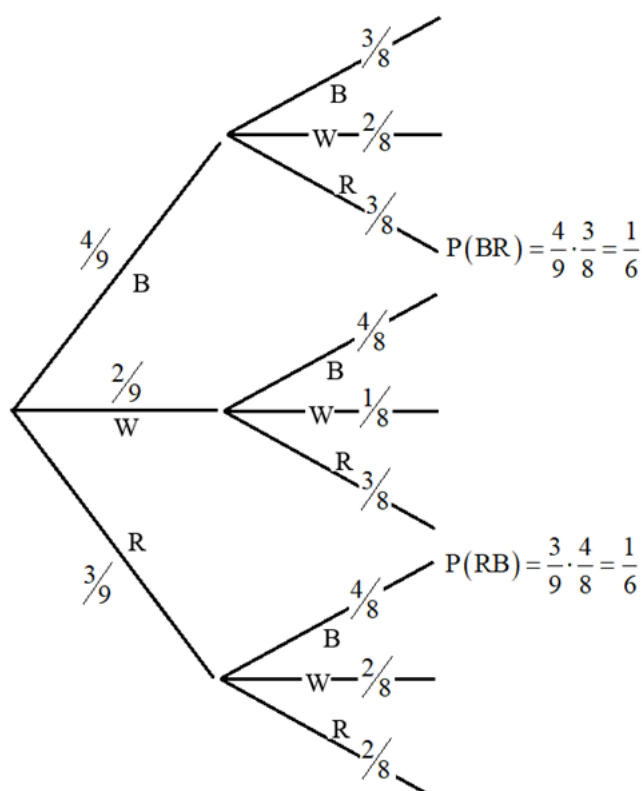


Quiz1 - Probability of Events SOLUTION KEY

[total marks on quiz: 35 marks]

No GDC allowed on questions 1-5; GDC is allowed on the last question #6.

1. A bag contains 4 black balls, 2 white balls and 3 red balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen. Find the probability of choosing one black ball and one red ball in any order. **[4 marks]**



probability of choosing BR or RB =

$$P(BR) + P(RB) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

2. Independent events A and B are such that $p(A) = 0.2$ and $p(A \cup B) = 0.6$. Find $p(B)$. **[4 marks]**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

since A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$

$$\text{thus, } P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$

letting $P(B) = x$ and substituting given values:

$$0.6 = 0.2 + x - 0.2x \Rightarrow 0.8x = 0.4 \Rightarrow x = \frac{0.4}{0.8} = \frac{1}{2} \quad \text{thus, } p(B) = \frac{1}{2}$$

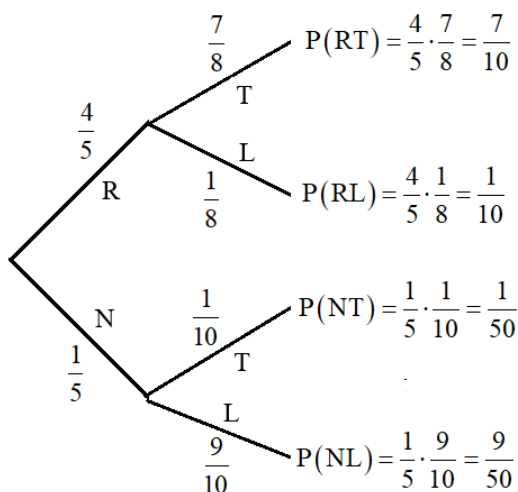
Quiz1 - Probability of Events SOLUTION KEY

3. An alarm clock is used to wake a student for school. The probability that the alarm rings is $\frac{4}{5}$.

If the alarm rings, there is a probability of $\frac{7}{8}$ that the student arrives at school on time; but, if the alarm does not ring, the probability that the student arrives at school on time is $\frac{1}{10}$.

Find:

- (a) the probability that the student arrives at school on time on a given day; **[4 marks]**
- (b) the probability that, on a randomly chosen morning on which the student is late for school, the alarm did not ring. **[4 marks]**



R = alarm rings; N = alarm does not ring

T = student is on time; L = student is late

- (a) probability student arrives on time =

$$P(RT) + P(NT) = \frac{7}{10} + \frac{1}{50} = \frac{35}{50} + \frac{1}{50} = \frac{36}{50} = \frac{18}{25} = 0.72$$

- (b) conditional probability:

$$P(N|L) = \frac{P(N \cap L)}{P(L)} = \frac{\frac{9}{50}}{\frac{1}{10} + \frac{9}{50}} = \frac{\frac{9}{50}}{\frac{14}{50}} = \frac{9}{14} \approx 0.643$$

4. Two dice are rolled. The score is the smaller of the two numbers that appear; if the same number appears on both dice, then the score is that number. What is the probability that the score is 3?

[4 marks]

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

The table at left shows the complete sample space.

probability score is 3 = $\frac{7}{36}$

Quiz1 - Probability of Events SOLUTION KEY

5. Events A and B are given such that $P(A \cap B) = \frac{1}{5}$, $P(B|A) = \frac{1}{2}$, $P(A|B) = \frac{3}{10}$. Find:

(a) $P(B)$; [3 marks]

(b) $P(A)$; [3 marks]

(c) $P(A \cup B)$ [2 marks]

$$(a) \quad P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow \frac{3}{10} = \frac{\frac{1}{5}}{P(B)} \Rightarrow P(B) = \frac{\frac{1}{5}}{\frac{3}{10}} = \frac{10}{15} = \frac{2}{3}$$

$$(b) \quad P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow \frac{1}{2} = \frac{\frac{1}{5}}{P(A)} \Rightarrow P(A) = \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{2}{5}$$

$$(c) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{5} + \frac{2}{3} - \frac{1}{5} = \frac{6}{15} + \frac{10}{15} - \frac{3}{15} = \frac{13}{15}$$

6. A couple is told that the probability that they will have blue-eyed children is $\frac{1}{4}$. The couple would like to have 6 children.

(a) What is the probability that 3 of the 6 children will be blue-eyed?

[4 marks]

(b) What is the probability that all 6 children will be blue-eyed?

[3 marks]

$$(a) \quad \text{probability that 3 of the 6 are blue-eyed} = \binom{6}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^3 \approx 0.132$$

$$(b) \quad \text{probability that all 6 are blue-eyed} = \left(\frac{1}{4}\right)^6 \approx 0.000244$$