Topic 1: Physics and physical measurement

1.2 Uncertainties and Errors

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As a result of a measurement, we get an approximate value of the measured quantity.

Consider, for example, the measurement of a pencil length with a ruler divided into millimetres.
- We cannot tell exactly where the ends are.
- We do not know how accurate the ruler scale is (instrumental uncertainty).
- We do not even know from which part of the right end the length should be measured (problem of definition).
The accuracy of a measurement device is called the instrumental uncertainty.

- The most obvious factor affecting the measurement accuracy is the measurement device used.
- You will find the instrumental uncertainty of many devices from an online manual. For example, the manuals of the Vernier instruments are at https://www.vernier.com/products/sensors/.
- In the absence of a manual, we may estimate the instrumental error to be the precision of the last digit in the device reading.
Example: Instrumental Uncertainty of Digital Multimeter in Measuring Temperature

- For example, according to the manual, the instrumental uncertainty of a Finest 205 Multimeter in measuring temperature is ±4°C, while the last digit rule gives ±1°C.

- In this case, the last digit rule underestimates the real uncertainty by a factor of four!

- In many other cases, such as in digital scales, the last digit rule is a reasonable estimate.
Uncertainty in Measurement

The uncertainty in a measurement is the overall uncertainty in a measurement considering all the error sources.

- First, to determine the uncertainty in a measurement, you need to determine the instrumental uncertainty of the measurement device used. This is the lower limit of the uncertainty in measurement.

- Second, you should consider the other factors affecting the measurement accuracy. The examples include the limit of reading of an analog scale, the effect of parallax, and sources of random and systematic errors.

- Third, the uncertainty in measurement is estimated based on the instrumental uncertainty and the other factors.
Definition of Random Error

Random errors are sources of uncertainties in a measurement whose effect can be reduced by a repeated experiment, and taking the average of the results.

- An example of a random error is the uncertainty in the measurement of falling time by manual timing. Due mainly to reaction times at starting and ending the measurement, the measured times vary around the real value of the falling time randomly. The uncertainty in manual timing is about $\pm 0.2s$.
- The measurement value may vary randomly especially in digital devices.
**Systematic Errors**

**Definition of Systematic Error**

Systematic errors are sources of uncertainties in the measurement that tend to shift the measurement values by a fixed amount. A repeated experiment and taking the average does not reduce the effect of a systematic error.

- For example, if the bathroom scale has a zero offset error of 0.2 kg, all the measured masses will be off by a fixed amount of 0.2 kg.
- Other examples of systematic errors are measuring the mass of a sample with a container, forgetting to add atmospheric pressure to measured gauge pressure values, poorly calibrated instruments, and systematic misreading of a scale.
- Systematic errors can be hard to detect.
- In some cases, the y intercept of a best fit line indicates the systematic error in the measurement.
Definition of Accuracy

An indication of how close a measured value is to the accepted value (a measure of correctness).

Or, in the absence of an accepted value

Definition of Accuracy

An indication of how close a measured value is to the real value of the measured quantity.

For example, if a repeated measurement for the acceleration due to gravity yields the values $9.80\text{ms}^{-2}, 9.84\text{ms}^{-2}, 9.76\text{ms}^{-2}$, and $9.82\text{ms}^{-2}$, the measurement results are accurate (the accepted value for the acceleration due to gravity being $9.81\text{ms}^{-2}$).
Definition of Precision

An indication of the agreement among a number of measurements made in the same way (a measure of exactness).

- For example, if a repeated measurement yields the values 1.235A, 1.232A, 1.229A, and 1.231A for a constant electric current, the measurement results are precise (spread of data is low).
- Or, if a repeated measurement for the falling time yields 0.69s, 0.24s, 0.47s, and 0.50s, the measurement results are not precise (spread of data is high).
- **Note that measurement values may be precise, but not accurate.** As an example, the measured values of 9.41ms^{-2}, 9.44ms^{-2}, 9.39ms^{-2}, and 9.38ms^{-2} for the acceleration due to gravity are precise, but not accurate.
Propagation of Error

- When measured values are used in calculations, the associated uncertainties affect the uncertainty in the calculated result. This is called the propagation of error.

- Most of the time, we need only two simple rules for our calculations.

**Propagation of Error in Multiplication and Division**

For multiplication, division and powers, percentage uncertainties add.

**Propagation of Error in Subtraction and Addition**

For addition and subtraction, absolute uncertainties add.
Example of Propagation of Error

Example

Example 1. To find the volume of a cube, a student measured its side as \((8.2 \pm 0.1)\) cm. Calculate the volume of the cube with its uncertainty.

**Answer.** \(l = 8.2\) cm, \(\Delta l = 0.1\) cm, \(V, \Delta V = ?\)

The volume of the cube is

\[
V = l^3 = (8.2\text{ cm})^3 \approx 551\text{ cm}^3.
\]

The percentage uncertainty in a side of the cube is

\[
\Delta l\% = \frac{\Delta l}{l} = \frac{0.1\text{ cm}}{8.2\text{ cm}} \approx 0.0122 = 1.22\%.
\]
Example of Propagation of Error

Example

In multiplication, percentage uncertainties add. The percentage uncertainty in the volume is

\[ \Delta V\% = 3 \times \Delta l\% = 3 \times 0.0122 = 0.0366 = 3.66\%. \]

The absolute uncertainty in the volume is thus

\[ \Delta V = \Delta V\% \times V = 0.0366 \times 551 \text{ cm}^3 \approx 20 \text{ cm}^3. \]

The volume with uncertainty is

\[ V = (550 \pm 20) \text{ cm}^3. \]

Note! The absolute uncertainty in the volume has to be rounded to one significant figure, and the precision of volume has to match that of uncertainty.
Example 2. A group of students studied free fall by dropping a tennis ball from a shelf. In a repeated measurement they found the average of falling times to be \((0.6 \pm 0.1)\) s. By using the propagation of error, calculate the height of the shelf with uncertainty.

Answer. \(t_{av} = 0.6\) s, \(\Delta t_{av} = 0.1\) s, \(g = 9.8\) m/s\(^{-2}\), \(h = ?\)

In free fall, the distance fallen is \(h = \frac{1}{2}gt_{av}^2\), where \(g = 9.8\) m/s\(^{-2}\) is the acceleration due to gravity, and \(t\) is falling time.

The falling height is

\[
h = \frac{1}{2}gt_{av}^2 = \frac{1}{2} \times 9.8\) m/s\(^{-2}\) \times (0.6\) s\(^2\) \approx 1.8\) m.
\]
Example of Propagation of Error

Example

The percentage uncertainty in falling time is

$$\Delta t_{av,\%} = \frac{\Delta t_{av}}{t_{av}} = \frac{0.1\text{s}}{0.6\text{s}} \approx 0.167 = 16.7\%. $$

The percentage uncertainty in falling time squared is

$$\Delta t^2_{av,\%} = 2 \times \Delta t_{av,\%} = 2 \times 0.167 = 0.334 = 33.4\%. $$

The absolute uncertainty in falling time squared is

$$\Delta t^2_{av} = \Delta t^2_{av,\%} \times t^2_{av} = 0.334 \times (0.6\text{s})^2 \approx 0.120\text{s}^2.$$
Example

Assuming that the uncertainty in the acceleration due to gravity is $\Delta g \approx 0$, the absolute uncertainty in falling height is

$$\Delta h = \frac{1}{2} g \Delta t_{av}^2 = \frac{1}{2} \times 9.8 \text{ms}^{-2} \times 0.120 \text{s}^2 \approx 0.6 \text{m}.$$ 

The falling height with uncertainty is

$$h = (1.8 \pm 0.6) \text{m}$$

Note! Because of the high percentage uncertainty in the measurement of time, the uncertainty in the calculated height becomes exceptionally large.