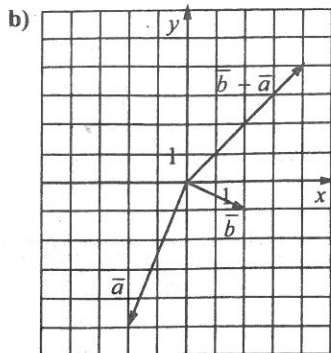


13. Ratkaisu a) $\vec{b} - \vec{a} = 2\vec{i} - \vec{j} - (-2\vec{i} - 5\vec{j}) = 2\vec{i} - \vec{j} + 2\vec{i} + 5\vec{j} = 4\vec{i} + 4\vec{j}$



c) Vektorien välinen kulma saadaan kaavalla $\cos(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$.

$$\cos(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-2 \cdot 2 + (-5) \cdot (-1)}{\sqrt{(-2)^2 + (-5)^2} \cdot \sqrt{2^2 + (-1)^2}}$$

$$= \frac{-4 + 5}{\sqrt{29} \cdot \sqrt{5}} = \frac{1}{\sqrt{29} \cdot \sqrt{5}} = 0,083045\dots$$

$$\sphericalangle(\vec{a}, \vec{b}) = 85,236\dots^\circ \approx 85,2^\circ$$

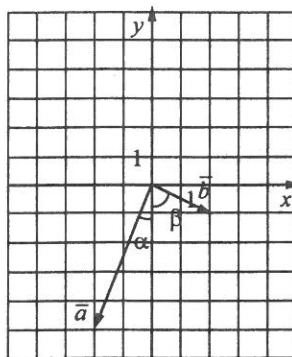
Vastaus a) $4\vec{i} + 4\vec{j}$ c) $85,2^\circ$

Toinen tapa c-kohdan kulman voi laskea myös geometrisesti:

$$\sphericalangle(\vec{a}, \vec{b}) = \alpha + \beta, \text{ missä } \tan \alpha = \frac{2}{5} \text{ ja } \tan \beta = 2.$$

Tällöin $\alpha \approx 21,801^\circ$ ja $\beta \approx 63,435^\circ$, joten

$$\sphericalangle(\vec{a}, \vec{b}) \approx 21,801^\circ + 63,435^\circ \approx 85,2^\circ.$$



A-osa

1. a) -1 b) 4,78 € c) 40

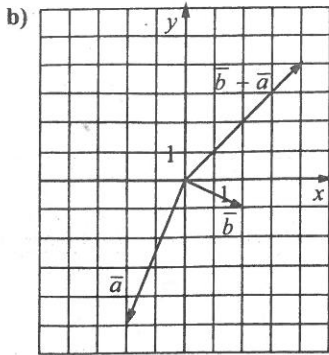
2. a) E b) E c) E d) E e) T f) T

3. i) 1 ja 2 ii) -1 iii) 0 iv) 2 v) $x = 1$ vi) -1

4. a) Hasse $10 \text{ €} / 40 \text{ €} = 0,25 = 25 \%$ Anette $9 \text{ €} / 30 \text{ €} = 0,3 = 30 \%$

b) $x < 2/3$

13. Ratkaisu a) $\vec{b} - \vec{a} = 2\vec{i} - \vec{j} - (-2\vec{i} - 5\vec{j}) = 2\vec{i} - \vec{j} + 2\vec{i} + 5\vec{j} = 4\vec{i} + 4\vec{j}$



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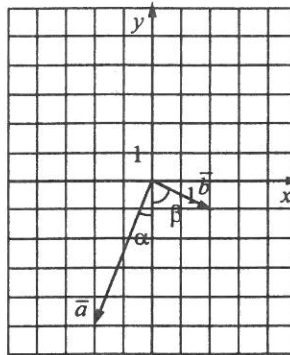
Vastaus a) $4\vec{i} + 4\vec{j}$ c) $85,2^\circ$

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A-osa

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