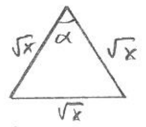
16.4 Poikkileikkaus:



$\alpha = 60^\circ$

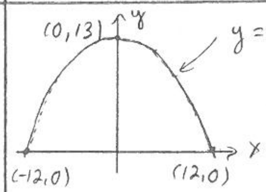
$A(x) = \frac{1}{2}\cdot\sqrt{x}\cdot\sqrt{x}\cdot\sin 60^\circ$

$= \frac{1}{2}x\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}x$

$$V = \int_0^{24} A(x)\,dx = \int_0^{24}\frac{\sqrt{3}}{4}x\,dx = \frac{\sqrt{3}}{4}\Big/_0^{24}\frac{1}{2}x^2 = \frac{\sqrt{3}}{4}\left(\frac{1}{2}\cdot 24^2 - 0\right)$$

$$= 72\sqrt{3} = 124{,}71 \approx \underline{125\ (m^3)}$$

16.5



$y = ax^2 + bx + c$

$$\begin{cases} (12,0):\ 12^2a + 12b + c = 0 & (1)\\ (-12,0):\ (-12)^2a - 12b + c = 0 & (2)\\ (0,13):\ \quad c = 13 & (3)\end{cases}$$

$(1)+(2):\ 288a + 2c = 0 \iff a = -\frac{2c}{288} = -\frac{2\cdot 13}{288} = -\frac{13}{144}$

$(1)-(2):\ 24b = 0 \iff b = 0$

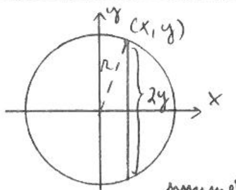
$\Rightarrow y = -\frac{13}{144}x^2 + 13$

$$A = \int_{-12}^{12}\left(-\frac{13}{144}x^2 + 13\right)dx \overset{\text{symmetria}}{=} 2\Big/_0^{12}\left(-\frac{13}{144}\cdot\frac{1}{3}x^3 + 13x\right)$$

$$= 2\left[\left(-\frac{13}{144}\cdot\frac{1}{3}\cdot 12^3 + 13\cdot 12\right) - 0\right] = 208\ (m^2)$$

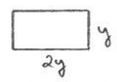
$V = Ah = 208\ m^2\cdot 42\ m = 8736\ m^3 \approx \underline{8740\ m^3}$

16.9



$x^2 + y^2 = r^2$, $2r = 19{,}7\,m$

Poikkileikkaus:



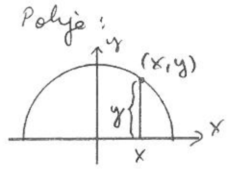
$A(x) = 2y\cdot y = 2y^2 = 2(r^2 - x^2)$

$$V = \int_{-r}^{r} A(x)\,dx \overset{\text{symmetria}}{=} 2\int_0^r 2(r^2 - x^2)\,dx = 4\Big/_0^r\left(r^2x - \frac{1}{3}x^3\right)$$

$$= 4\left[\left(r^2 r - \frac{1}{3}r^3\right) - 0\right] = 4\cdot\frac{2}{3}r^3 = \frac{8}{3}\cdot\left(\frac{19{,}7\,m}{2}\right)^3$$

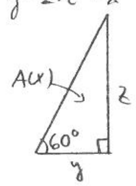
$$= 2548{,}46\,m^3 \approx \underline{2550\,m^3}$$

16.12 Pohja:



$x^2 + y^2 = r^2 \iff y^2 = r^2 - x^2$

Poikkileikkaus:



$\tan 60^\circ = \frac{z}{y} = \sqrt{3}$

$\iff z = \sqrt{3}\,y$

$A(x) = \frac{1}{2}yz = \frac{1}{2}y\sqrt{3}y = \frac{\sqrt{3}}{2}y^2 = \frac{\sqrt{3}}{2}(r^2 - x^2)$

$$V = \int_{-r}^{r} A(x)\,dx \overset{\text{symmetria}}{=} 2\int_0^r \frac{\sqrt{3}}{2}(r^2 - x^2)\,dx = \sqrt{3}\Big/_0^r\left(r^2x - \frac{1}{3}x^3\right)$$

$$= \sqrt{3}\left[\left(r^2 r - \frac{1}{3}r^3\right) - 0\right] = \sqrt{3}\cdot\frac{2}{3}r^3 = \frac{2\sqrt{3}}{3}(8{,}0\,m)^3$$

$$= 591{,}207\,m^3 \approx \underline{590\,m^3}$$