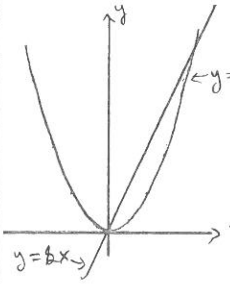
13.18



Tarkastellaan tilannetta $k>0$ (symmetria)

$\begin{cases} y=x^2 \\ y=kx \end{cases} \Rightarrow x^2=kx \Leftrightarrow x^2-kx=0 \Leftrightarrow x(x-k)=0 \Leftrightarrow x=\begin{cases}0\\k\end{cases}$

$A=\int_0^k (kx-x^2)\,dx = \Big/_0^k \left(k\cdot\frac{1}{2}x^2-\frac{1}{3}x^3\right)$

$=\left(k\cdot\frac{1}{2}k^2-\frac{1}{3}k^3\right)-0=\frac{1}{6}k^3=10\frac{2}{3} \quad |\cdot 6$

$\Leftrightarrow k^3=64 \quad |\sqrt[3]{\ } \quad \Leftrightarrow k=4 \Rightarrow$ symmetria: $\underline{k=\pm 4}$

14.1 $f(x)=2x-3 \quad , x\in[0,5]$

$V=\pi\int_0^5 (2x-3)^2\,dx=\pi\cdot\frac{1}{2}\int_0^5 2(2x-3)^2\,dx=\frac{\pi}{2}\Big/_0^5 \frac{1}{3}(2x-3)^3$

$=\frac{\pi}{2}\cdot\frac{1}{3}\left[(2\cdot5-3)^3-(2\cdot0-3)^3\right]=\underline{\frac{185}{3}\pi}$

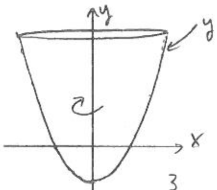
14.3 $f(x)=4x^3-9 \quad , -2\le x\le 2$

$V=\pi\int_{-2}^2 (4x^3-9)^2\,dy=\pi\int_{-2}^2(16x^6-72x^3+81)\,dx$

$=\pi\Big/_{-2}^2\left(16\cdot\frac{1}{7}x^7-72\cdot\frac{1}{4}x^4+81x\right)$

$=\pi\left[\left(\frac{16}{7}\cdot2^7-18\cdot2^4+81\cdot2\right)-\left(\frac{16}{7}\cdot(-2)^7-18\cdot(-2)^4+81\cdot(-2)\right)\right]=\underline{\frac{6364}{7}\pi}$

14.5



$y=x^2-1 \Leftrightarrow x^2=y+1$

$x=0: \ y=0^2-1=-1$

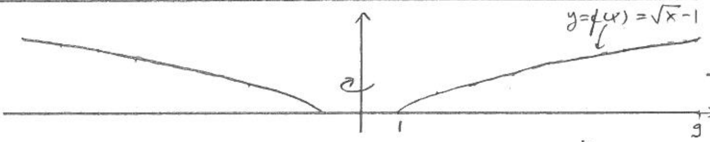
$x=2: \ y=2^2-1=3$

$V=\pi\int_{-1}^3 x^2\,dy=\pi\int_{-1}^3(y+1)\,dy$

$=\pi\Big/_{-1}^3\left(\frac{1}{2}y^2+y\right)=\pi\left[\left(\frac{1}{2}\cdot3^2+3\right)-\left(\frac{1}{2}\cdot(-1)^2-1\right)\right]=\underline{8\pi}$

14.7 a) 5 b) 3 c) 6 d) 2

14.13



$y=\sqrt{x}-1 \Leftrightarrow \sqrt{x}=y+1 \quad |()^4 \Leftrightarrow x^2=(y+1)^4$

$x=1: \ y=\sqrt{1}-1=0 \quad ; \ x=9: \ y=\sqrt{9}-1=2$

$V=\pi\int_0^2 x^2\,dy=\pi\int_0^2(y+1)^4\,dy=\pi\Big/_0^2\frac{1}{5}(y+1)^5$

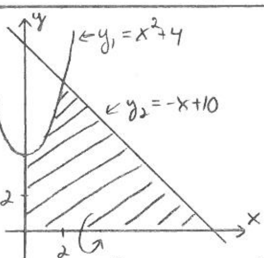
$=\pi\left[\frac{1}{5}(2+1)^5-\frac{1}{5}(0+1)^5\right]=\pi\left[\frac{1}{5}\cdot3^5-\frac{1}{5}\right]=\underline{\frac{242}{5}\pi}$

14.19 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad |\cdot b^2 \Leftrightarrow y^2=b^2-\frac{b^2}{a^2}x^2$

$V=\pi\int_{-a}^a y^2\,dx \overset{\text{symmetria}}{=} 2\pi\int_0^a\left(b^2-\frac{b^2}{a^2}x^2\right)dx=2\pi\Big/_0^a\left(b^2x-\frac{b^2}{a^2}\cdot\frac{1}{3}x^3\right)$

$=2\pi\left[\left(b^2a-\frac{b^2}{a^2}\cdot\frac{1}{3}a^3\right)-0\right]=2\pi\cdot\frac{2}{3}b^2a=\underline{\frac{4}{3}\pi b^2a}$

15.1



Leikkauskohdat:

$\begin{cases}y_1=x^2+4\\y_2=-x+10\end{cases}$

$\Rightarrow x^2+4=-x+10 \Leftrightarrow x^2+x-6=0 \Leftrightarrow x=\begin{cases}-3\\2\end{cases}$

0-kohta: $-x+10=0 \Leftrightarrow x=10$

$V_1=\pi\int_0^2 y_1^2\,dx=\pi\int_0^2(x^2+4)^2\,dx=\pi\int_0^2(x^4+8x^2+16)\,dx$

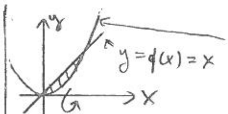
$=\pi\Big/_0^2\left(\frac{1}{5}x^5+8\cdot\frac{1}{3}x^3+16x\right)=\pi\left[\left(\frac{1}{5}\cdot2^5+\frac{8}{3}\cdot2^3+16\cdot2\right)-0\right]=\frac{896}{15}\pi$

$V_2=\pi\int_2^{10} y_2^2\,dx=\pi\int_2^{10}(-x+10)^2\,dx=-\pi\int_2^{10}-(-x+10)^2\,dx=-\pi\Big/_2^{10}\frac{1}{3}(-x+10)^3$

$=-\pi\left[\frac{1}{3}(-10+10)^3-\frac{1}{3}(-2+10)^3\right]=\frac{512}{3}\pi$ [TAI: $V_2=\frac{1}{3}\pi\cdot8^2\cdot8=\frac{512}{3}\pi$ (kartio)]

$V=V_1+V_2=\underline{\frac{1152}{15}\pi}$

15.2



$y=g(x)=x^2$ Leikkauskohdat:

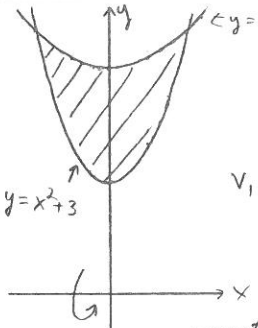
$\begin{cases}y=x^2\\y=x\end{cases} \Rightarrow x^2=x \Leftrightarrow x^2-x=0 \Leftrightarrow x=\begin{cases}0\\1\end{cases}$

$V_1=\pi\int_0^1 y^2\,dx=\pi\int_0^1 x^2\,dx=\pi\Big/_0^1\frac{1}{3}x^3=\pi\left(\frac{1}{3}\cdot1^3-\frac{1}{3}\cdot0^3\right)=\frac{\pi}{3}$

$V_2=\pi\int_0^1 y^2\,dx=\pi\int_0^1(x^2)^2\,dx=\pi\int_0^1 x^4\,dx=\pi\Big/_0^1\frac{1}{5}x^5=\pi\left(\frac{1}{5}\cdot1^5-\frac{1}{5}\cdot0^5\right)=\frac{\pi}{5}$

$\Rightarrow V=V_1-V_2=\frac{\pi}{3}-\frac{\pi}{5}=\underline{\frac{2\pi}{15}}$

15.4



Leikkauskohdat:

$\begin{cases}y=\frac{1}{4}x^2+6\\y=x^2+3\end{cases} \Rightarrow x^2+3=\frac{1}{4}x^2+6 \Leftrightarrow 3x^2=12 \Leftrightarrow x^2=4 \Leftrightarrow x=\pm2$

$V_1=\pi\int_{-2}^2\left(\frac{1}{4}x^2+6\right)^2dx \overset{\text{symm.}}{=} 2\pi\int_0^2\left(\frac{1}{16}x^4+3x^2+36\right)$

$=2\pi\Big/_0^2\left(\frac{1}{16}\cdot\frac{1}{5}x^5+3\cdot\frac{1}{3}x^3+36x\right)$

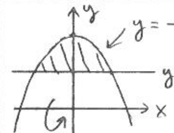
$=2\pi\left[\left(\frac{1}{80}\cdot2^5+2^3+36\cdot2\right)-0\right]=\frac{804}{5}\pi$

$V_2=\pi\int_{-2}^2(x^2+3)^2\,dx \overset{\text{symmetria}}{=} 2\pi\int_0^2(x^4+6x^2+9)\,dx=2\pi\Big/_0^2\left(\frac{1}{5}x^5+6\cdot\frac{1}{3}x^3+\dots\right)$

$=2\pi\left[\left(\frac{1}{5}\cdot2^5+2\cdot2^3+9\cdot2\right)-0\right]=\frac{404}{5}\pi$

$V=V_1-V_2=\frac{804}{5}\pi-\frac{404}{5}\pi=\underline{80\pi}$

15.5



Leikkauskohdat: $\begin{cases}y=-x^2+2\\y=1\end{cases}$

$\Rightarrow -x^2+2=1 \Leftrightarrow x^2=1 \quad |\sqrt{\ } \Leftrightarrow x=\pm1$

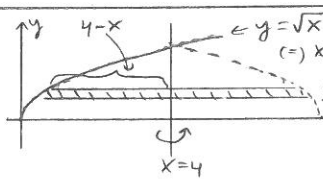
$V_1=\pi\int_{-1}^1(-x^2+2)^2\,dx \overset{\text{symmetria}}{=} 2\pi\int_0^1(x^4-4x^2+4)\,dx=2\pi\Big/_0^1\left(\frac{1}{5}x^5-4\cdot\frac{1}{3}x^3+\dots\right)$

$=2\pi\left[\left(\frac{1}{5}\cdot1^5-\frac{4}{3}\cdot1^3+4\cdot1\right)-0\right]=\frac{86}{15}\pi$

$V_2=\pi\cdot1^2\cdot2=2\pi$ (lieriö)

$\Rightarrow V=V_1-V_2=\frac{86}{15}\pi-2\pi=\underline{\frac{56}{15}\pi}$

15.16



Viipaleen:
- paksuus: dy
- säde: $4-x=4-y^2$
- tilavuus: $\pi(4-y^2)^2dy$

$x=4: \ y=\sqrt{4}=2$

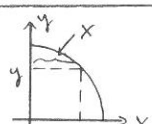
$V=\int_0^2\pi(4-y^2)^2\,dy=\pi\int_0^2(16-8y^2+y^4)\,dy$

$=\pi\Big/_0^2\left(16y-8\cdot\frac{1}{3}y^3+\frac{1}{5}y^5\right)=\pi\left[\left(16\cdot2-\frac{8}{3}\cdot2^3+\frac{1}{5}\cdot2^5\right)-0\right]=\underline{\frac{256}{15}\pi}$

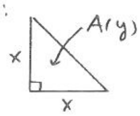
16.1 $V=\int_0^{1,4}A(x)\,dx=\int_0^{1,4}\left(\sqrt{7-5x}\right)^2dx=\int_0^{1,4}(7-5x)\,dx$

$=\Big/_0^{1,4}\left(7x-5\cdot\frac{1}{2}x^2\right)=\left(7\cdot1,4-\frac{5}{2}\cdot1,4^2\right)-0=4,9\,(m^3)\Rightarrow \underline{4900\,l}$

16.2



Poikkileikkaus:



Kuution särmä: $a=12,0$ cm

$x^2+y^2=a^2 \Leftrightarrow x^2=a^2-y^2$

$V=\int_0^a A(y)\,dy=\int_0^a\frac{1}{2}\cdot x\cdot x\,dy=\int_0^a\frac{1}{2}x^2\,dy=\int_0^a\frac{1}{2}(a^2-y^2)\,dy$

$=\frac{1}{2}\Big/_0^a\left(a^2y-\frac{1}{3}y^3\right)=\frac{1}{2}\left[\left(a^2a-\frac{1}{3}a^3\right)-0\right]$

$=\frac{1}{2}\cdot\frac{2}{3}a^3=\frac{1}{3}a^3=\frac{1}{3}(12,0\text{ cm})^3=\underline{576\text{ cm}^3}$