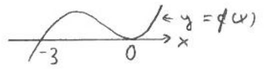
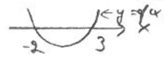
11.11 a) $f(x) = x^3 + 3x^2 = x^2(x+3) = 0 \Leftrightarrow x = \begin{cases} 0 \\ -3 \end{cases}$

Testipiste: $f(-1) = 2 > 0$



$A = \int_{-3}^{0} (x^3+3x^2)\,dx = \Big/_{-3}^{0} \tfrac{1}{4}x^4 + x^3 = 0 - (\tfrac{1}{4}\cdot(-3)^4 + (-3)^3) = \underline{\tfrac{27}{4}}$

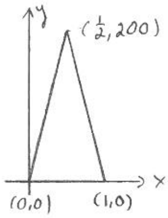
b) $g(x) = x^2 - x - 6 = 0 \Leftrightarrow x = \begin{cases} 3 \\ -2 \end{cases}$



$A = -\int_{-2}^{3} (x^2-x-6)\,dx = -\Big/_{-2}^{3} \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 6x$

$= -\left[(\tfrac{1}{3}\cdot 3^3 - \tfrac{1}{2}\cdot 3^2 - 6\cdot 3) - (\tfrac{1}{3}\cdot(-2)^3 - \tfrac{1}{2}\cdot(-2)^2 - 6\cdot(-2))\right] = \underline{\tfrac{125}{6}}$

11.21



$k_1 = \frac{\Delta y}{\Delta x} = \frac{200-0}{\frac{1}{2}-0} = 400$

$y - 0 = 400(x-0) \Leftrightarrow y = 400x$

$k_2 = \frac{\Delta y}{\Delta x} = \frac{0-200}{1-\frac{1}{2}} = -400$

$y - 0 = -400(x-1) \Leftrightarrow y = -400x + 400$

$\Rightarrow f(x) = \begin{cases} 400x, & 0 \le x \le \frac{1}{2} \\ -400x + 400, & \frac{1}{2} < x \le 1 \end{cases}$

$\begin{cases} \lim_{x\to\frac{1}{2}-} f(x) = \lim_{x\to\frac{1}{2}-} 400x = 400\cdot\frac{1}{2} = 200 = f(\frac{1}{2}) \\ \lim_{x\to\frac{1}{2}+} f(x) = \lim_{x\to\frac{1}{2}+} (-400x+400) = -400\cdot\frac{1}{2} + 400 = 200 \end{cases}$

$\Rightarrow f$ jatkuva kohdassa $x = \frac{1}{2}$ $\Rightarrow f$ jatkuva väl. $[0,1]$

$\int_0^1 f(x)\,dx = \frac{1}{2}\cdot 1\cdot 200 = 100$

12.1 a) $A = \int_1^3 x\,dy = \int_1^3 \sqrt{y}\,dy = \int_1^3 y^{\frac{1}{2}}\,dy = \Big/_1^3 \tfrac{2}{3} y^{\frac{3}{2}}$

$= \frac{2}{3}\cdot 3^{\frac{3}{2}} - \frac{2}{3}\cdot 1^{\frac{3}{2}} = \frac{2}{3}3\sqrt{3} - \frac{2}{3} = \underline{2\sqrt{3} - \frac{2}{3}}$ $(\approx 2{,}80)$

b) $A = \int_0^3 x\,dy = \int_0^3 2^y\,dy = \Big/_0^3 \frac{1}{\ln 2} 2^y = \frac{1}{\ln 2}\cdot 2^3 - \frac{1}{\ln 2}\cdot 2^0$

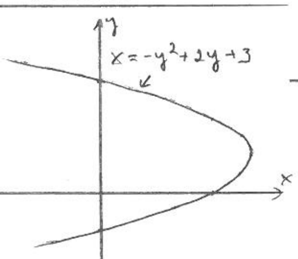
$= \frac{1}{\ln 2}\cdot 8 - \frac{1}{\ln 2}\cdot 1 = \underline{\frac{7}{\ln 2}}$ $(\approx 10{,}10)$

12.2

y	$x = -y^2+2y+3$
0	3
1	4
2	3
3	0
4	-5
-1	0
-2	-5

$x = -y^2 + 2y + 3 = 0$

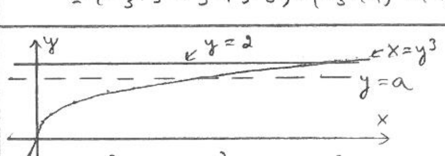
$\Leftrightarrow y = \begin{cases} 3 \\ -1 \end{cases}$



$A = \int_{-1}^{3} x\,dy = \int_{-1}^{3} (-y^2+2y+3)\,dy = \Big/_{-1}^{3} -\tfrac{1}{3}y^3 + 2\cdot\tfrac{1}{2}y^2 + 3y$

$= (-\frac{1}{3}\cdot 3^3 + 3^2 + 3\cdot 3) - (-\frac{1}{3}\cdot(-1)^3 + (-1)^2 + 3\cdot(-1)) = 9 - (-\frac{5}{3}) = \underline{\frac{32}{3}}$

12.3

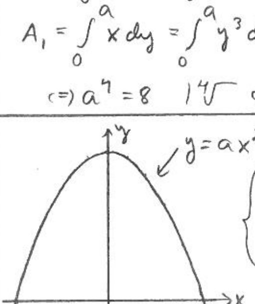


$A = \int_0^2 x\,dy = \int_0^2 y^3\,dy = \Big/_0^2 \tfrac{1}{4}y^4 = \tfrac{1}{4}\cdot 2^4 - \tfrac{1}{4}\cdot 0^4 = 4$

$A_1 = \int_0^a x\,dy = \int_0^a y^3\,dy = \Big/_0^a \tfrac{1}{4}y^4 = \tfrac{1}{4}a^4 - 0 = \tfrac{1}{4}a^4 = \tfrac{1}{2}A = 2 \quad |\cdot 4$

$\Leftrightarrow a^4 = 8 \quad |\sqrt[4]{\ } \quad \Leftrightarrow a = (\pm)\sqrt[4]{8} \Rightarrow \underline{y = \sqrt[4]{8}}$ $(\approx 1{,}68)$

12.5



$\begin{cases} (0,4): & c = 4 & (1) \\ (-2{,}5;0): & (-2{,}5)^2 a - 2{,}5b + c = 0 & (2) \\ (2{,}5;0): & 2{,}5^2 a + 2{,}5b + c = 0 & (3) \end{cases}$

$(2)+(3): \ 12{,}5a + 8 = 0 \Leftrightarrow a = -0{,}64$

$(3)-(2): \ 5b = 0 \Leftrightarrow b = 0$

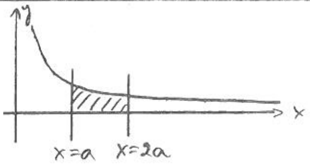
$\Rightarrow y = -0{,}64x^2 + 4$

$A = \int_{-2{,}5}^{2{,}5} (-0{,}64x^2+4)\,dx = 2\Big/_0^{2{,}5} (-0{,}64\cdot\tfrac{1}{3}x^3 + 4x) = 2\left[(-0{,}64\cdot\tfrac{1}{3}\cdot 2{,}5^3 + 4\cdot 2{,}5) - 0\right]$

$\approx 13{,}3333 \approx \underline{13{,}3\,(m^2)}$

12.6 a) 3 b) 1 c) 2 d) 4

12.10

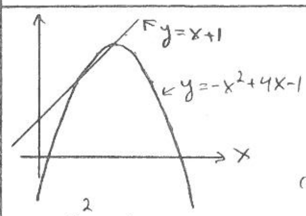


$A = \int_a^{2a} \frac{1}{x}\,dx = \Big/_a^{2a} \ln|x|$

$= \ln 2a - \ln a = \ln\frac{2a}{a}$

$= \ln 2$ (vakio) $\Rightarrow$ väite

13.1



Leikkauskohdat

$\begin{cases} y = x+1 \\ y = -x^2+4x-1 \end{cases}$

$\Rightarrow x + 1 = -x^2 + 4x - 1$

$\Leftrightarrow x^2 - 3x + 2 = 0 \Leftrightarrow x = \begin{cases} 1 \\ 2 \end{cases}$

$A = \int_1^2 ((-x^2+4x-1) - (x+1))\,dx = \int_1^2 (-x^2+3x-2)\,dx$

$= \Big/_1^2 (-\tfrac{1}{3}x^3 + 3\cdot\tfrac{1}{2}x^2 - 2x) = (-\tfrac{1}{3}\cdot 2^3 + \tfrac{3}{2}\cdot 2^2 - 2\cdot 2) - (-\tfrac{1}{3}\cdot 1^3 + \tfrac{3}{2}\cdot 1^2 - 2\cdot 1)$

$= \underline{\tfrac{1}{6}}$

13.2 b) Leikkauskohdat: $\begin{cases} y = 0{,}2x^2 + 1 \\ y = -x + 13 \end{cases}$

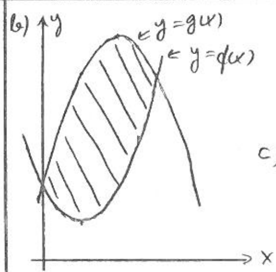
$\Rightarrow 0{,}2x^2 + 1 = -x + 13 \Leftrightarrow 0{,}2x^2 + x - 12 = 0 \Leftrightarrow x \approx \begin{cases} -10{,}6394 \\ 5{,}63941 \end{cases}$

$A = \int_{-5{,}7}^{5{,}63941} ((-x+13) - (0{,}2x^2+1))\,dx = \Big/_{-5{,}7}^{5{,}63941} -0{,}2\cdot\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + 12x$

$= (-\frac{0{,}2}{3}\cdot 5{,}63941^3 - \frac{1}{2}\cdot 5{,}63941^2 + 12\cdot 5{,}63941) - (-\frac{0{,}2}{3}\cdot(-5{,}7)^3 - \frac{1}{2}\cdot(-5{,}7)^2 + 12\cdot(-5{,}7))$

$\approx 111{,}965 \Rightarrow 111{,}965\cdot(10\,m)^2 \approx 11196{,}5\,m^2 \approx \underline{1{,}1\,ha}$

13.8 b)



a) $f(x) = g(x)$

$\Leftrightarrow x^2 - 2x + 2 = -x^2 + 4x + 2$

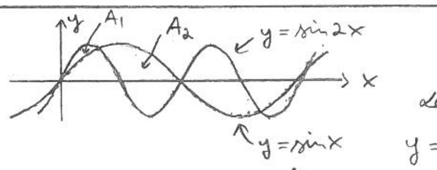
$\Leftrightarrow 2x^2 - 6x = 0 \Leftrightarrow x = \begin{cases} 0 \\ 3 \end{cases}$

c) $\int_0^3 (g(x) - f(x))\,dx$

$= \int_0^3 (-2x^2+6x)\,dx = \Big/_0^3 -2\cdot\tfrac{1}{3}x^3 + 6\cdot\tfrac{1}{2}x^2$

$= (-\frac{2}{3}\cdot 3^3 + 3\cdot 3^2) - 0 = \underline{9}$

13.10



Leikkauskohdat: $y = \sin 2x = \sin x$

$\Leftrightarrow 2x = x + n2\pi$ tai $2x = \pi - x + n2\pi$

$\Leftrightarrow x = n2\pi$ tai $3x = \pi + n2\pi \quad |:3$

$\Leftrightarrow x = \frac{\pi}{3} + n\frac{2\pi}{3}, \quad n \in \mathbb{Z}$

2 peräkkäistä erikokoista aluetta: $0, \frac{\pi}{3}, \pi$

$A_1 = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\,dx = \Big/_0^{\frac{\pi}{3}} -\tfrac{1}{2}\cos 2x + \cos x$

$= (-\frac{1}{2}\cos\frac{2\pi}{3} + \cos\frac{\pi}{3}) - (-\frac{1}{2}\cos 0 + \cos 0)$

$= (-\frac{1}{2}\cdot(-\frac{1}{2}) + \frac{1}{2}) - (-\frac{1}{2}\cdot 1 + 1) = \frac{1}{4}$

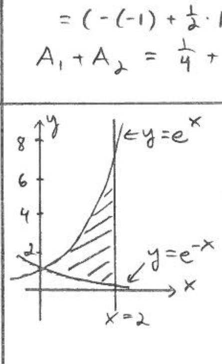
$A_2 = \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x)\,dx = \Big/_{\frac{\pi}{3}}^{\pi} -\cos x + \tfrac{1}{2}\cos 2x$

$= (-\cos\pi + \frac{1}{2}\cos 2\pi) - (-\cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3})$

$= (-(-1) + \frac{1}{2}\cdot 1) - (-\frac{1}{2} + \frac{1}{2}\cdot(-\frac{1}{2})) = \frac{9}{4}$

$A_1 + A_2 = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \underline{\frac{5}{2}}$

13.11



Leikkauskohdat:

$\begin{cases} y = e^x \\ y = e^{-x} \end{cases} \Rightarrow e^x = e^{-x} \Leftrightarrow x = -x$

$\Leftrightarrow 2x = 0 \Leftrightarrow x = 0$

$A = \int_0^2 (e^x - e^{-x})\,dx = \Big/_0^2 (e^x + e^{-x})$

$= (e^2 + e^{-2}) - (e^0 + e^0)$

$= \underline{e^2 + e^{-2} - 2}$ $(\approx 5{,}52)$