c) $\int x^2 e^{-x^3}dx = -\frac{1}{3}\int -3x^2 e^{-x^3}dx = \underline{-\frac{1}{3}e^{-x^3}+C}$

d) $\int 2e^{x^2}dx$ ei onnistu, ei saada sisäfunktion derivaattaa mukaan $(D x^2 = 2x)$

8.18

$$\int_0^{\ln 2}(e^{2x}+2e^{-x})dx = \int_0^{\ln 2} e^{2x}dx + \int_0^{\ln 2} 2e^{-x}dx$$
$$= \frac{1}{2}\int_0^{\ln 2} 2e^{2x}dx + 2\cdot(-1)\int_0^{\ln 2} -e^{-x}dx = \frac{1}{2}\Big/_0^{\ln 2} e^{2x} - 2\Big/_0^{\ln 2} e^{-x}$$
$$= \frac{1}{2}[e^{2\ln 2}-e^0] - 2[e^{-\ln 2}-e^0]$$
$$= \frac{1}{2}[e^{\ln 2^2}-1] - 2[e^{\ln 2^{-1}}-1] = \frac{1}{2}(2^2-1) - 2(2^{-1}-1)$$
$$= \frac{1}{2}(4-1) - 2\left(\frac{1}{2}-1\right) = \frac{1}{2}\cdot 3 - 2\cdot\left(-\frac{1}{2}\right) = \frac{3}{2}+1 = \underline{\frac{5}{2}}$$

9.1

a) $\int 7\sin x\,dx = 7\int \sin x\,dx = \underline{-7\cos x + C}$

b) $\int \sin 6x\,dx = \frac{1}{6}\int 6\sin 6x\,dx = \frac{1}{6}(-\cos 6x)+C = \underline{-\frac{1}{6}\cos 6x + C}$

c) $\int \sin(-x)\,dx = -\int -\sin(-x)\,dx = -(-\cos(-x))+C = \underline{\cos(-x)+C}$

$(= \cos x + C$, kosini on parillinen funktio$)$

[TAI: $\int \sin(-x)\,dx = \int -\sin x\,dx = \cos x + C$]

d) $\int -\frac{\sin x}{3}dx = -\frac{1}{3}\int \sin x\,dx = -\frac{1}{3}(-\cos x)+C = \underline{\frac{1}{3}\cos x + C}$

9.3

a) $\int \frac{1}{2}\cos x\,dx = \frac{1}{2}\int \cos x\,dx = \frac{1}{2}\sin x + C$

b) $\int_{\frac{\pi}{2}}^{\pi}\cos 2x\,dx = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} 2\cos 2x\,dx = \frac{1}{2}\Big/_{\frac{\pi}{2}}^{\pi}\sin 2x$

$= \frac{1}{2}\left(\sin(2\pi) - \sin\left(2\cdot\frac{\pi}{2}\right)\right) = \frac{1}{2}(0-0) = \underline{0}$

9.6

a) $\int \cos x \sin^2 x\,dx = \int \underbrace{\cos x}_{f'(x)}(\underbrace{\sin x}_{f(x)})^2dx = \frac{1}{3}(\sin x)^3 + C = \underline{\frac{1}{3}\sin^3 x + C}$

b) $\int \sin x \cos^3 x\,dx = -\int -\sin x(\cos x)^3dx = -\frac{1}{4}(\cos x)^4 + C = \underline{-\frac{1}{4}\cos^4 x + C}$

9.8

Anna:

$\int \sin 2x\,dx$
$= \frac{1}{2}\int 2\sin 2x\,dx$
$= \frac{1}{2}\cdot(-\cos 2x) + C$ ←
$= -\frac{1}{2}\cos 2x + C$

Ben:

$\int \sin 3x\,dx$
$= \frac{1}{3}\int 3\sin 3x\,dx$ ←
$= \frac{1}{3}(-\cos 3x) + C$
$= -\frac{1}{3}\cos 3x + C$

9.9

a) $\int \frac{2\cos x}{\sin^2 x}dx = \int 2\cos x(\sin x)^{-2}dx = 2\int \cos x(\sin x)^{-2}dx$

$= 2\cdot\frac{1}{-1}(\sin x)^{-1}+C = \underline{-\frac{2}{\sin x}+C,\ 0<x<\pi}$

b) $\int \frac{\cos x}{3\sqrt{\sin x}}dx = \frac{1}{3}\int \cos x(\sin x)^{-\frac{1}{2}}dx = \frac{1}{3}\cdot\frac{1}{\frac{1}{2}}(\sin x)^{\frac{1}{2}}+C$

$= \underline{\frac{2}{3}\sqrt{\sin x}+C,\ 0<x<\pi}$

9.18

a) $\int \cos^2 x\,dx = \int\left(\frac{1}{2}\cos 2x + \frac{1}{2}\right)dx \qquad (\cos 2x = 2\cos^2 x - 1)$

$= \int \frac{1}{2}\cos 2x\,dx + \int \frac{1}{2}dx = \frac{1}{2}\cdot\frac{1}{2}\int 2\cos 2x\,dx + \frac{1}{2}x$

$= \underline{\frac{1}{4}\sin 2x + \frac{1}{2}x + C}$

b) $\int(\cos^2 x - \sin^2 x)dx = \int \cos 2x\,dx = \frac{1}{2}\int 2\cos 2x\,dx$

$= \underline{\frac{1}{2}\sin 2x + C}$

c) $\int(\cos x - \sin x)^2dx = \int(\cos^2 x - 2\sin x\cos x + \sin^2 x)dx$

$= \int(1-\sin 2x)dx = x - \frac{1}{2}\int 2\sin 2x\,dx$

$= x - \frac{1}{2}(-\cos 2x) + C = \underline{x + \frac{1}{2}\cos 2x + C}$

10.2

$f(x) = x^2 + \frac{2}{x} + \frac{1}{x^2}, \quad x<0$

$F(x) = \int\left(x^2 + \frac{2}{x} + \frac{1}{x^2}\right)dx = \int\left(x^2 + 2\cdot\frac{1}{x} + x^{-2}\right)dx$

$= \frac{1}{3}x^3 + 2\ln|x| + \frac{1}{-1}x^{-1} + C = \underline{\frac{1}{3}x^3 + 2\ln(-x) - \frac{1}{x} + C,\ x<0}$

10.3

a) $\int \frac{1}{x+3}dx = \ln|x+3| + C = \underline{\ln(x+3)+C,\ x>0}$

b) $\int \frac{1}{3x+1}dx = \frac{1}{3}\int\frac{3}{3x+1}dx = \frac{1}{3}\ln|3x+1|+C = \underline{\frac{1}{3}\ln(3x+1)+C,\ x>0}$

c) $\int \frac{4x}{x^2+4}dx = 4\cdot\frac{1}{2}\int\frac{2x}{x^2+4}dx = 2\ln|x^2+4|+C = \underline{2\ln(x^2+4)+C,\ x>0}$

10.4

a) $\int \frac{2x}{x^2+1}dx = \ln|\underbrace{x^2}_{\geq 0}+1| + C = \underline{\ln(x^2+1)+C}$

b) $\int \frac{2}{x^2+1}dx$ ei onnistu, ei saada alakerran derivaattaa ylös $(D(x^2+1) = 2x)$

c) $\int \frac{x}{x^2+1}dx = \frac{1}{2}\int\frac{2x}{x^2+1}dx = \frac{1}{2}\ln|x^2+1|+C = \underline{\frac{1}{2}\ln(x^2+1)}$

d) $\int \frac{2x}{(x^2+1)^2}dx = \int 2x(x^2+1)^{-2}dx = \frac{1}{-1}(x^2+1)^{-1}+C = \underline{-\frac{1}{x^2+1}+}$

10.5

$f(x) = \frac{1}{2x-1}, \quad x \neq \frac{1}{2}$

$F(x) = \int\frac{1}{2x-1}dx = \frac{1}{2}\int\frac{2}{2x-1}dx = \frac{1}{2}\ln|2x-1| + C$

$= \begin{cases}\frac{1}{2}\ln(-(2x-1)) + C = \frac{1}{2}\ln(1-2x) + C, & x<\frac{1}{2}\\ \frac{1}{2}\ln(2x-1) + D, & x>\frac{1}{2}\end{cases}$

$F(0) = \frac{1}{2}\ln 1 + C = C = -1$

$F(1) = \frac{1}{2}\ln 1 + D = D = 4$ $\Rightarrow F(x) = \begin{cases}\frac{1}{2}\ln(1-2x)-1, & x<\frac{1}{2}\\ \frac{1}{2}\ln(2x-1)+4, & x>\end{cases}$

10.8

a) $\int_0^1\frac{dx}{5x+1} = \frac{1}{5}\int_0^1\frac{5}{5x+1}dx = \frac{1}{5}\Big/_0^1\ln|5x+1| = \frac{1}{5}(\ln 6 - \underbrace{\ln 1}_{=0})$

$= \underline{\frac{1}{5}\ln 6}$

b) $\int_1^0\frac{x^2}{x^3+1}dx = \frac{1}{3}\int_1^0\frac{3x^2}{x^3+1}dx = \frac{1}{3}\Big/_1^0\ln|x^3+1|$

$= \frac{1}{3}(\underbrace{\ln 1}_{=0} - \ln 2) = \underline{-\frac{1}{3}\ln 2}$

c) $\int_0^1\frac{e^x}{e^x+1}dx = \Big/_0^1\ln|e^x+1| = \ln(e^1+1) - \ln(e^0+1)$

$= \ln(e+1) - \ln 2 = \underline{\ln\frac{e+1}{2}}$

10.15

a) $\int\frac{dx}{3x+3} = \frac{1}{3}\int\frac{3}{3x+3}dx = \frac{1}{3}\ln|3x+3|+C = \underline{\frac{1}{3}\ln(3x+3)+C,\ x}$

b) $\int\frac{dx}{(3x+3)^2} = \frac{1}{3}\int 3(3x+3)^{-2}dx = \frac{1}{3}\cdot\frac{1}{-1}(3x+3)^{-1}+C$

$= \underline{-\frac{1}{3(3x+3)}+C,\ x>-1}$

c) $\int\frac{dx}{\sqrt{3x+3}} = \frac{1}{3}\int 3(3x+3)^{-\frac{1}{2}}dx = \frac{1}{3}\cdot\frac{1}{\frac{1}{2}}(3x+3)^{\frac{1}{2}}+C$

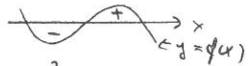
$= \underline{\frac{2}{3}\sqrt{3x+3}+C}$

11.1

$f(x) = -3x^3 + 12x = 3x(-x^2+4) = 0 \Leftrightarrow x = \begin{cases}0\\ \pm 2\end{cases}$

Testipisteet: $f(-1) = -9 < 0$

$f(1) = 9 > 0$



$A = A_1 + A_2 = -\int_{-2}^0(-3x^3+12x)dx + \int_0^2(-3x^3+12x)dx$

$= -\Big/_{-2}^0 -3\cdot\frac{1}{4}x^4 + 12\cdot\frac{1}{2}x^2 + \Big/_0^2 -3\cdot\frac{1}{4}x^4 + 12\cdot\frac{1}{2}x^2$

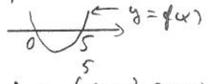
$= -\left[0 - \left(-\frac{3}{4}\cdot(-2)^4 + 6\cdot(-2)^2\right)\right] + \left[\left(-\frac{3}{4}\cdot 2^4 + 6\cdot 2^2\right) - 0\right]$

$= 12 + 12 = \underline{24}$

11.3

$f(x) = 6x^2 - 30x$

a) $f(x) = 6x^2 - 30x = 6x(x-5) = 0 \Leftrightarrow x = \begin{cases}0\\ 5\end{cases}$

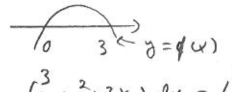


b) $A = -\int_0^5(6x^2-30x)dx = -\Big/_0^5 6\cdot\frac{1}{3}x^3 - 30\cdot\frac{1}{2}x^2$

$= -[(2\cdot 5^3 - 15\cdot 5^2) - 0] = \underline{125}$

11.4

$f(x) = -x^2 + 3x = x(-x+3) = 0 \Leftrightarrow x = \begin{cases}0\\ 3\end{cases}$



$A = \int_0^3(-x^2+3x)dx = \Big/_0^3 -\frac{1}{3}x^3 + 3\cdot\frac{1}{2}x^2 = \left(-\frac{1}{3}\cdot 3^3 + \frac{3}{2}\cdot 3^2\right) - 0 = \frac{9}{2}$

11.7

a) 2 b) 3 c) 4 d) 1