5.2 a) $\int_1^3 (x^3+2x)\,dx = \Big/_1^3 \frac14 x^4 + x^2 = (\frac14\cdot 3^4+3^2) - (\frac14\cdot 1^4+1^2) = \underline{28}$

b) $\int_{-2}^4 (3x^2-x+1)\,dx = \Big/_{-2}^4 x^3 - \frac12 x^2 + x$

$= (4^3-\frac12\cdot 4^2+4) - ((-2)^3 - \frac12\cdot(-2^2)-2) = 60-(-12) = \underline{72}$

5.3 $A=\int_{-2}^1 (x^2+1)\,dx = \Big/_{-2}^1 \frac13 x^3 + x = (\frac13\cdot 1^3+1) - (\frac13\cdot(-2)^3 - 2) = \frac43 - (-\frac{14}{3}) = \underline{6}$

5.8 $f(x) = \int_2^x (2t^3-3t^2)\,dt = \Big/_2^x 2\cdot\frac14 t^4 - 3\cdot\frac13 t^3 = \Big/_2^x \frac12 t^4 - t^3$

$= (\frac12 x^4 - x^3) - (\frac12\cdot 2^4 - 2^3) = \frac12 x^4 - x^3 = x^3(\frac12 x - 1) = 0$

$\Leftrightarrow x^3 = 0$ tai $\frac12 x - 1 = 0 \Leftrightarrow \underline{x=0 \text{ tai } x=2}$

5.13 $A = \int_{-1}^2 (\frac14 x^3 + 2)\,dx = \Big/_{-1}^2 \frac14\cdot\frac14 x^4 + 2x = \Big/_{-1}^2 \frac1{16}x^4+2x$

$= (\frac1{16}\cdot 2^4 + 2\cdot 2) - (\frac1{16}\cdot(-1)^4 + 2\cdot(-1)) = 5 - (-\frac{31}{16}) = \underline{\frac{111}{16}}\ (\approx 6{,}94)$

5.17 $\int_a^{a+1}(2x+3)\,dx = \Big/_a^{a+1} x^2+3x = ((a+1)^2+3(a+1)) - (a^2+3a)$

$= a^2+2a+1+3a+3-a^2-3a = 2a+4 = \frac12 \Leftrightarrow \underline{a=-\frac74}$

6.1 a) $\int \frac1{x^2}\,dx = \int x^{-2}\,dx = \frac1{-1}x^{-1}+C = \underline{-\frac1x + C},\ x>0$

b) $\int\frac1{\sqrt x}\,dx = \int x^{-\frac12}\,dx = \frac{1}{\frac12}x^{\frac12}+C = \underline{2\sqrt x + C},\ x>0$

c) $\int 2x\sqrt x\,dx = \int 2x\cdot x^{\frac12}\,dx = \int 2x^{\frac32}\,dx = 2\cdot\frac{1}{\frac52}\underbrace{x^{\frac52}}_{x^{2+\frac12}}+C$

$= \underline{\frac45 x^2\sqrt x + C}$

6.5 $f(x)=\frac1{x^4},\ x\ne0$

a) $F(x) = \int\frac1{x^4}\,dx = \int x^{-4}\,dx = \frac1{-3}x^{-3}+C = -\frac1{3x^3}+C$

$\Rightarrow F(x) = \begin{cases} -\frac1{3x^3}+C, & x<0\\ -\frac1{3x^3}+D, & x>0\end{cases}$

b) $F(-1)=1 \Leftrightarrow -\frac{1}{3(-1)^3}+C = \frac13 + C = 1 \Leftrightarrow C=\frac23$

$F(1)=0 \Leftrightarrow -\frac{1}{3\cdot1^3}+D = -\frac13+D=0\Leftrightarrow D=\frac13$

$\Rightarrow F(x) = \begin{cases} -\frac1{3x^3}+\frac23, & x<0\\ -\frac1{3x^3}+\frac13, & x>0\end{cases}$

6.9 $A=\int_1^4\frac1{\sqrt x}\,dx = \int_1^4 x^{-\frac12}\,dx = \Big/_1^4 \frac1{\frac12}x^{\frac12} = \Big/_1^4 2\sqrt x$

$= 2\sqrt4 - 2\sqrt1 = 2\cdot2-2\cdot1 = \underline{2}$

6.13 $f(x)=\sqrt x - \frac1{\sqrt x}$

$F(x) = \int(\sqrt x - \frac1{\sqrt x})\,dx = \int(x^{\frac12}-x^{-\frac12})\,dx = \frac23 x^{\frac32} - 2x^{\frac12}+C$

$= \underline{\frac23 x\sqrt x - 2\sqrt x + C},\ x>0$

6.17 $A = \int_1^2 (x\sqrt x+1)\,dx = \int_1^2(x^1\cdot x^{\frac12}+1)\,dx = \int_1^2(x^{\frac32}+1)\,dx$

$= \Big/_1^2 \frac25 x^{\frac52}+x = (\frac25\cdot \underbrace{2^{\frac52}}_{2^{2+\frac12}}+2) - (\frac25\cdot1^{\frac52}+1)$

$= \frac25\cdot2^2\sqrt2+2-\frac25-1 = \underline{\frac85\sqrt2+\frac35}\ (\approx 2{,}86)$

7.1 a) $\int x^2(x^3-5)^4\,dx = \frac13\int3x^2(x^3-5)^4\,dx = \frac13\cdot\frac15(x^3-5)^5+C$

$= \underline{\frac1{15}(x^3-5)^5+C}$

b) $\int x(x^2+1)^5\,dx = \frac12\int2x(x^2+1)^5\,dx = \frac12\cdot\frac16(x^2+1)^6+C$

$= \underline{\frac1{12}(x^2+1)^6+C}$

7.2 a) $\int\sqrt{3-x}\,dx = -\int -(3-x)^{\frac12}\,dx = -\frac23(3-x)^{\frac32}+C$

$= \underline{-\frac23(3-x)\sqrt{3-x}+C},\ x<3$

b) $\int x\sqrt{x^2+1}\,dx = \frac12\int2x(x^2+1)^{\frac12}\,dx = \frac12\cdot\frac23(x^2+1)^{\frac32}+$

$= \underline{\frac13(x^2+1)\sqrt{x^2+1}+C}$

7.5 a) $\int_1^2\sqrt{5x-1}\,dx = \frac15\int_1^2 5(5x-1)^{\frac12}\,dx = \frac15\Big/_1^2\frac23(5x-1)^{\frac32}$

$= \frac2{15}[(5\cdot2-1)^{\frac32} - (5\cdot1-1)^{\frac32}] = \frac2{15}[(\sqrt9)^3-(\sqrt4)^3]$

$= \frac2{15}[3^3-2^3] = \frac2{15}(27-8) = \underline{\frac{38}{15}}$

b) $\int_1^2\frac1{\sqrt{5x-1}}\,dx = \int_1^2(5x-1)^{-\frac12}\,dx = \frac15\int_1^2 5(5x-1)^{-\frac12}\,dx$

$= \frac15\Big/_1^2 2(5x-1)^{\frac12} = \frac25\Big/_1^2\sqrt{5x-1} = \frac25[\sqrt{5\cdot2-1}-\sqrt{5\cdot1-1}]$

$= \frac25(\sqrt9-\sqrt4) = \frac25(3-2) = \underline{\frac25}$

7.6 a) $\int 2(x^2-1)^5\,dx$ ei onnistu, ei saada sisäfunktion derivaattaa mukaan ($D(x^2-1)=2x$)

b) $\int x(x^2-1)^3\,dx = \frac12\int2x(x^2-1)^3\,dx = \frac12\cdot\frac14(x^2-1)^4+C$

$= \underline{\frac18(x^2-1)^4+C}$

c) $\int x^2(x^2-1)^4\,dx$ ei onnistu, ei saada sisäfunktion derivaattaa mukaan ($D(x^2-1)=2x$)

7.7 $A=\int_0^2\sqrt{2-x}\,dx = -\int_0^2-(2-x)^{\frac12}\,dx = -\Big/_0^2\frac23(2-x)^{\frac32}$

$= -\frac23[(2-2)^{\frac32}-(2-0)^{\frac32}] = -\frac23(0-2\sqrt2) = \underline{\frac{4\sqrt2}{3}}\ (\approx 1{,}8$

7.15 a) $\int(x+2)\sqrt{x^2+4x}\,dx = \frac12\int2(x+2)(x^2+4x)^{\frac12}\,dx$

$=\frac12\cdot\frac23(x^2+4x)^{\frac32}+C = \underline{\frac13(x^2+4x)\sqrt{x^2+4x}+C},\ x>0$

b) $\int\frac{4x+1}{\sqrt{4x^2+2x}}\,dx = \frac12\int2(4x+1)(4x^2+2x)^{-\frac12}\,dx$

$= \frac12\cdot2(4x^2+2x)^{\frac12}+C = \underline{\sqrt{4x^2+2x}+C},\ x>0$

8.1 a) $\int\frac{2e^x}{5}\,dx = \frac25\int e^x\,dx = \frac25e^x+C$

b) $\int e^{6x}\,dx = \frac16\int6e^{6x}\,dx = \frac16e^{6x}+C$

c) $\int e^{-x}\,dx = -\int-e^{-x}\,dx = -e^{-x}+C$

d) $\int e^{\frac x9}\,dx = \int e^{\frac19x}\,dx = 9\int\frac19e^{\frac19x}\,dx = 9e^{\frac19x}+C$

8.2 a) $De^{8x} = e^{8x}\cdot 8$

b) $\int e^{8x}\,dx = \frac18\int8e^{8x}\,dx = \frac18e^{8x}+C$

8.5 $f(x)=1-e^{2x}$

$F(x) = \int(1-e^{2x})\,dx = \int1\,dx - \int e^{2x}\,dx = x - \frac12\int2e^{2x}\,dx$

$= x-\frac12e^{2x}+C$, F jatk. ja deriv. R:ssä

$F'(x)=f(x)=1-e^{2x}=0 \Leftrightarrow e^{2x}=1=e^0\Leftrightarrow 2x=0\Leftrightarrow x=0$

		0	
F'	+		−
F	↗	max	↘

$F'(-1)\approx0{,}86>0$, $F'(1)\approx-6{,}4<0$

suurin arvo: $F(0) = 0-\frac12e^0+C = -\frac12+C = \frac32$

$\Leftrightarrow C=2 \Rightarrow \underline{F(x)=x-\frac12e^{2x}+2}$

8.8 $\int_0^{\ln2}(e^x-e^{-x})\,dx = \Big/_0^{\ln2}(e^x+e^{-x}) = (e^{\ln2}+e^{-\ln2}) - (e^0+e^0$

$= (2+e^{\ln2^{-1}}) - (1+1) = e^{\ln2^{-1}} = 2^{-1} = \underline{\frac12}$

8.11 a) $\int e^{-5x}\,dx = -\frac15\int-5e^{-5x}\,dx = \underline{-\frac15e^{-5x}+C}$

b) $\int e^{x^3}\,dx$ ei onnistu, ei saada sisäfunktion derivaattaa mukaan ($Dx^3=3x^2$)