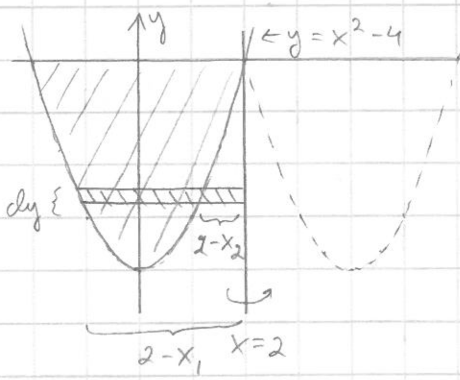
9.



$y(0) = 0^2-4 = -4$

$y = x^2-4 \iff x^2 = y+4 \;|\sqrt{\;} \iff x = \pm\sqrt{y+4}$

Iso kappale:

$V_1 = \pi\int_{-4}^{0}(2-x_1)^2\,dy = \pi\int_{-4}^{0}(2-(-\sqrt{y+4}))^2\,dy$

$= \pi\int_{-4}^{0}(2+\sqrt{y+4})^2\,dy = \pi\int_{-4}^{0}(4+4\overbrace{\sqrt{y+4}}^{(y+4)^{\frac12}}+(y+4))\,dy$

$= \pi \Big/_{-4}^{0}(8y+\frac12 y^2+4\cdot\frac23(y+4)^{\frac32}) = \pi\left[(0+0+\frac83(\sqrt4)^3)-(8\cdot(-4)+\frac12(-4)^2+\frac83\cdot 0)\right]$

$= \frac{136}{3}\pi$

Kolo:

$V_2 = \pi\int_{-4}^{0}(2-x_2)^2\,dy = \pi\int_{-4}^{0}(2-\sqrt{y+4})^2\,dy = \pi\int_{-4}^{0}(4-4\sqrt{y+4}+(y+4))\,dy$

$= \pi \Big/_{-4}^{0}(8y+\frac12 y^2-4\cdot\frac23(y+4)^{\frac32}) = \pi\left[(0+0-\frac43\cdot(\sqrt4)^3-(8\cdot(-4)+\frac12(-4)^2-\frac83\cdot 0)\right]$

$= \frac83\pi$

Tilavuus: $V = V_1 - V_2 = \frac{136}{3}\pi - \frac83\pi = \underline{\frac{128}{3}\pi}$

MAA 7 24.11.2023

1. $f(x) = -2x+6$

$F(x) = \int(-2x+6)\,dx = -2\cdot\frac12 x^2+6x+C$

a) $F(1) = -1^2+6\cdot 1+C = 5+C = -2 \iff C = -7 \Rightarrow \underline{F(x) = -x^2+6x-7}$

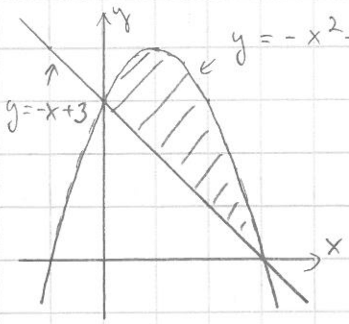
b) F jatk. ja deriv. R:ssä, $F'(x) = f(x) = -2x+6 = 0 \iff x = 3$

F'	+	−
F	↗	↘
	3 max	

suurin arvo: $F(3) = -3^2+6\cdot 3+C = 5 \iff C = -4$

$\Rightarrow \underline{F(x) = -x^2+6x-4}$

2.



$y = -x^2+2x+3$ Leikkauskohdat: $\begin{cases} y = -x^2+2x+3 \\ x+y-3 = 0 \iff y = 3-x \end{cases}$

$\Rightarrow 3-x = -x^2+2x+3 \iff x^2-3x = 0 \iff x = \begin{cases}0\\3\end{cases}$

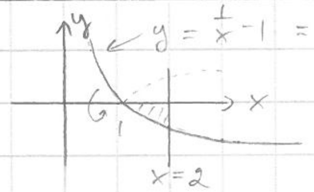
$A = \int_0^3((-x^2+2x+3)-(3-x))\,dx = \int_0^3(-x^2+3x)\,dx = \Big/_0^3 -\frac13x^3+3\cdot\frac12x^2$

$= (-\frac13\cdot 3^3+\frac32\cdot 3^2)-(-\frac13\cdot 0^3+\frac32\cdot 0^2) = \underline{\frac92}$

2'. $f(x) = \int_0^x(3t^2-4t-3)\,dt = \Big/_0^x 3\cdot\frac13t^3-4\cdot\frac12t^2-3t = (x^3-2x^2-3x)-0 = 0$

$\iff x(x^2-2x-3) = 0 \iff x = 0$ tai $x^2-2x-3 = 0 \iff \underline{x = 0 \text{ tai } x = \begin{cases}-1\\3\end{cases}}$

3.



$y = \frac1x - 1 = 0 \quad |\cdot x \iff 1-x = 0 \iff x = 1$

$V = \pi\int_1^2(\frac1x-1)^2\,dx = \pi\int_1^2(\frac{1}{x^2}-\frac2x+1)\,dx = \pi\int_1^2(x^{-2}-\frac2x+1)\,dx$

$= \pi\Big/_1^2(\frac{1}{-1}x^{-1}-2\ln|x|+x) = \pi\left[(-\frac12-2\ln2+2)-(-\frac11-2\underbrace{\ln1}_{=0}+1)\right]$

$= \underline{(\frac32-2\ln2)\pi}$

3'. a) $\int_0^3\frac{2}{(3x+1)^2}\,dx = \int_0^3 2(3x+1)^{-2}\,dx = 2\cdot\frac13\int_0^3 3(3x+1)^{-2}\,dx = \frac23\Big/_0^3\frac{1}{-1}(3x+1)^{-1}$

$= \frac23\left[-\frac{1}{3\cdot3+1}-(-\frac{1}{3\cdot0+1})\right] = \frac23(-\frac1{10}+1) = \frac23\cdot\frac9{10} = \underline{\frac35}$

b) $\int_0^4\frac{1}{\sqrt{2x+1}}\,dx = \int_0^4(2x+1)^{-\frac12}\,dx = \frac12\int_0^4 2(2x+1)^{-\frac12}\,dx = \frac12\Big/_0^4\frac{1}{\frac12}(2x+1)^{\frac12}$

$= \sqrt{2\cdot4+1}-\sqrt{2\cdot0+1} = 3-1 = \underline{2}$

4. a) $\int_0^{\sqrt{\ln2}}5xe^{x^2}\,dx = 5\cdot\frac12\int_0^{\sqrt{\ln2}}2xe^{x^2}\,dx = \frac52\Big/_0^{\sqrt{\ln2}}e^{x^2}$

$= \frac52\left[e^{(\sqrt{\ln2})^2}-e^{0^2}\right] = \frac52(e^{\ln2}-1) = \frac52(2-1) = \underline{\frac52}$

b) $\int_0^\pi(\cos2x+\sin3x)\,dx = \Big/_0^\pi\frac12\sin2x-\frac13\cos3x$

$= (\frac12\underbrace{\sin2\pi}_{=0}-\frac13\underbrace{\cos3\pi}_{=-1})-(\frac12\underbrace{\sin0}_{=0}-\frac13\underbrace{\cos0}_{=1}) = \frac13+\frac13 = \underline{\frac23}$