4. a) $f(x) = 3e^{-\frac{x}{5}} - 1 = 0 \iff 3e^{-\frac{x}{5}} = 1 \quad |:3 \iff e^{-\frac{x}{5}} = \frac{1}{3} \quad |\ln$

$\iff -\frac{x}{5} = \ln\frac{1}{3} = \ln 1 - \ln 3 = -\ln 3 \quad |\cdot(-5) \iff x = 5\ln 3 \ (\approx 5{,}49)$

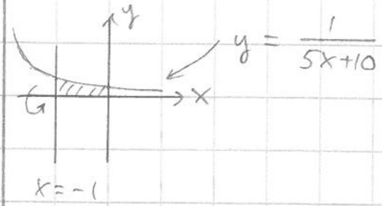
b)

$$A = \int_0^{5\ln 3} (3e^{-\frac{x}{5}} - 1)\,dx = \Big/_0^{5\ln 3} (3\cdot(-5)e^{-\frac{x}{5}} - x)$$

$$= (-15e^{-\frac{5\ln 3}{5}} - 5\ln 3) - (-15e^{0} - 0)$$

$$= (-15e^{\ln 3^{-1}} - 5\ln 3) + 15 = -15\cdot\frac{1}{3} - 5\ln 3 + 15 = \underline{10 - 5\ln 3} \ (\approx 4{,}51)$$

5.



a) $A = \int_{-1}^{0} \frac{1}{5x+10}\,dx = \frac{1}{5}\int_{-1}^{0} \frac{5}{5x+10}\,dx = \frac{1}{5}\Big/_{-1}^{0} \ln|5x+10|$

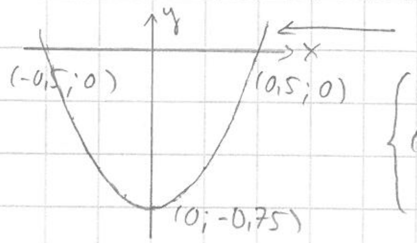
$= \frac{1}{5}\left[\ln(5\cdot 0+10) - \ln(5\cdot(-1)+10)\right] = \frac{1}{5}(\ln 10 - \ln 5)$

$= \frac{1}{5}\ln\frac{10}{5} = \underline{\frac{1}{5}\ln 2}$

b) $V = \pi\int_{-1}^{0}\left(\frac{1}{5x+10}\right)^2 dx = \pi\int_{-1}^{0}\frac{1}{(5x+10)^2}\,dx = \pi\int_{-1}^{0}(5x+10)^{-2}\,dx = \pi\cdot\frac{1}{5}\int_{-1}^{0} 5(5x+10)^{-2}\,dx$

$= \frac{\pi}{5}\Big/_{-1}^{0}\frac{1}{-1}(5x+10)^{-1} = \frac{\pi}{5}\left[-\frac{1}{5\cdot 0+10} - \left(-\frac{1}{5\cdot(-1)+10}\right)\right] = \frac{\pi}{5}\left(-\frac{1}{10}+\frac{1}{5}\right) = \frac{\pi}{5}\cdot\frac{1}{10} = \underline{\frac{\pi}{50}}$

6.



$y = f(x) = ax^2 + bx + c$

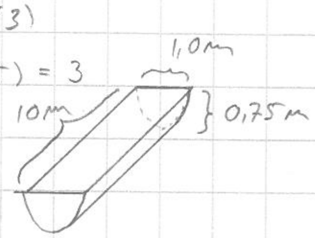
$$\begin{cases} (0{,}5;0): & a\cdot 0{,}5^2 + b\cdot 0{,}5 + c = 0 \\ (-0{,}5;0): & a\cdot(-0{,}5)^2 + b\cdot(-0{,}5) + c = 0 \\ (0;-0{,}75): & c = -0{,}75 \end{cases}$$

$$\iff \begin{cases} 0{,}25a + 0{,}5b + c = 0 & (1) \\ 0{,}25a - 0{,}5b + c = 0 & (2) \\ c = -0{,}75 & (3) \end{cases}$$

$(1) + (2): \ 0{,}5a + 2c = 0 \iff a = -4c = -4\cdot(-0{,}75) = 3$

$(1) - (2): \ b = 0$

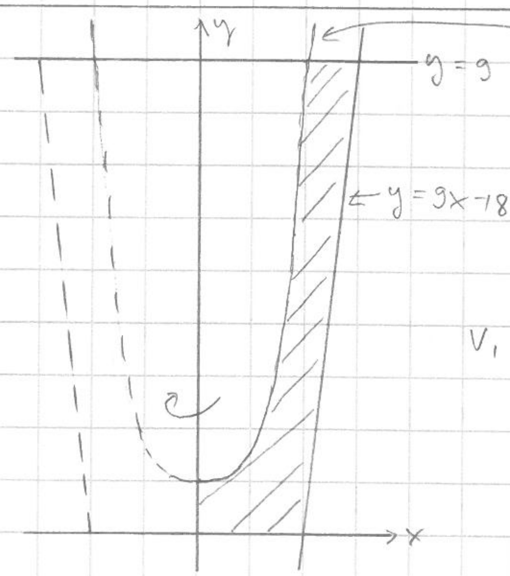
$\Rightarrow y = 3x^2 - 0{,}75$



$$A = -\int_{-0,5}^{0,5}(3x^2 - 0{,}75)\,dx \overset{\text{symmetria}}{=} -2\Big/_0^{0,5} 3\cdot\frac{1}{3}x^3 - 0{,}75x = -2\left[(0{,}5^3 - 0{,}75\cdot 0{,}5) - 0\right] = 0{,}5\,(m^2)$$

$\Rightarrow V = Ah = 0{,}5\,m^2\cdot 10\,m = \underline{5\,m^3}$

7.



$y = \frac{1}{2}x^4 + 1 \qquad y(0) = \frac{1}{2}\cdot 0^4 + 1 = 1$

$y = \frac{1}{2}x^4 + 1 \quad |\cdot 2 \iff x^4 = 2y - 2 \quad |\sqrt{\ }$

$\iff x^2 = \sqrt{2y-2}$

$y = 9x - 18 \quad |:9 \iff x = \frac{1}{9}y + 2$

Iso kappale:

$V_1 = \pi\int_0^9 x^2\,dy = \pi\int_0^9\left(\frac{1}{9}y + 2\right)^2 dx = \pi\int_0^9\left(\frac{1}{81}y^2 + \frac{4}{9}y + 4\right)dy$

$= \pi\Big/_0^9 \frac{1}{81}\cdot\frac{1}{3}y^3 + \frac{4}{9}\cdot\frac{1}{2}y^2 + 4y$

$= \pi\left[\left(\frac{1}{243}\cdot 9^3 + \frac{2}{9}\cdot 9^2 + 4\cdot 9\right) - 0\right] = 57\pi$

Kolo:

$V_2 = \pi\int_1^9 x^2\,dy = \pi\int_1^9 \sqrt{2y-2}\,dy = \pi\int_1^9 (2y-2)^{\frac{1}{2}}\,dy = \frac{\pi}{2}\int_1^9 2(2y-2)^{\frac{1}{2}}\,dy$

$= \frac{\pi}{2}\Big/_1^9 \frac{2}{3}(2y-2)^{\frac{3}{2}} = \frac{\pi}{3}\left[(2\cdot 9-2)^{\frac{3}{2}} - (2\cdot 1-2)^{\frac{3}{2}}\right] = \frac{\pi}{3}\left((\sqrt{16})^3 - 0\right) = \frac{64}{3}\pi$

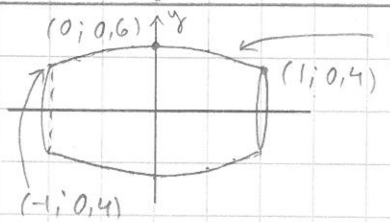
Lasin tilavuus: $V = V_1 - V_2 = 57\pi - \frac{64}{3}\pi = \frac{107}{3}\pi \ (cm^3)$

Lasin massa: $m = \rho V = 3120\,\frac{kg}{(100\,cm)^3}\cdot\frac{107}{3}\pi\,cm^3 \approx 0{,}3496\,kg \approx \underline{350\,g}$

7'. a) $A(t) = \int_0^t \frac{1}{12}x^2\,dx = \Big/_0^t \frac{1}{12}\cdot\frac{1}{3}x^3 = \frac{1}{36}t^3 - \frac{1}{36}\cdot 0^3 = \underline{\frac{1}{36}t^3\ (m^2)}$

b) $V = \int_0^6 A(t)\,dt = \int_0^6 \frac{1}{36}t^3\,dt = \Big/_0^6 \frac{1}{36}\cdot\frac{1}{4}t^4 = \frac{1}{144}\cdot 6^4 - 0 = \underline{9\ (m^3)}$

8.



$y = ax^2 + bx + c$

$$\begin{cases} (1;0{,}4): & a + b + c = 0{,}4 & (1) \\ (-1;0{,}4): & a - b + c = 0{,}4 & (2) \\ (0;0{,}6): & c = 0{,}6 & (3) \end{cases}$$

$(1) + (2): \ 2a + 2c = 0{,}8 \ |:2 \iff a = 0{,}4 - 0{,}6 = -0{,}2$

$(1) - (2): \ 2b = 0 \iff b = 0$

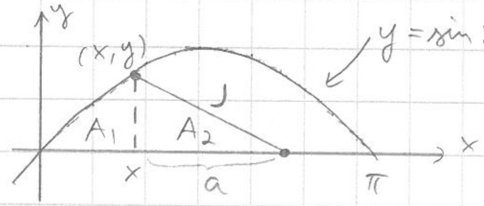
$\Rightarrow y = -0{,}2x^2 + 0{,}6$

$$V = \pi\int_{-1}^{1}(-0{,}2x^2 + 0{,}6)^2\,dx \overset{\text{symmetria}}{=} 2\pi\int_0^1 (0{,}04x^4 - 0{,}24x^2 + 0{,}36)\,dx$$

$$= 2\pi\Big/_0^1\left(0{,}04\cdot\frac{1}{5}x^5 - 0{,}24\cdot\frac{1}{3}x^3 + 0{,}36x\right) = 2\pi\left[\left(0{,}04\cdot\frac{1}{5}\cdot 1^5 - 0{,}24\cdot\frac{1}{3}\cdot 1^3 + 0{,}36\cdot 1\right) - 0\right]$$

$= 0{,}576\pi \approx 1{,}809557\ (m^3) \Rightarrow$ tilavuus $\underline{n.\ 1800\,l}$

10.



$A_1 = \int_0^x \sin t\,dt = \Big/_0^x -\cos t = -\cos x - (-\underbrace{\cos 0}_{=1}) = 1 - \cos x$

$k = \frac{\Delta y}{\Delta x} = \frac{0 - \sin x}{a} = -\frac{\sin x}{a} = -\frac{1}{2} \iff a = 2\sin x$

$A_2 = \frac{1}{2}\cdot a\cdot y = \frac{1}{2}\cdot 2\sin x\cdot\sin x = \sin^2 x$

$\Rightarrow \underline{A(x) = A_1 + A_2 = 1 - \cos x + \sin^2 x, \quad 0 < x < \pi}$

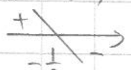
$= 1 - \cos x + (1 - \cos^2 x) = 2 - \cos x - \cos^2 x$

Mer. $t = \cos x$: $f(t) = 2 - t - t^2, \quad -1 \le t \le 1$

A:lla ja f:llä on samat arvojoukot, f jatk. ja deriv. väl. $[-1,1]$

$f'(t) = -1 - 2t = 0 \iff t = -\frac{1}{2}$

	-1		$-\frac{1}{2}$ max		1
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	



$\cos x = -\frac{1}{2} = \cos\frac{2\pi}{3} \iff x = \pm\frac{2\pi}{3} + n2\pi, \ n \in \mathbb{Z}$

$x \in [0,\pi] \Rightarrow \underline{x = \frac{2\pi}{3}}$