

$$\begin{aligned}
 \text{b) } \int \frac{1}{\sqrt{2x+1}} dx &= \int \frac{1}{(2x+1)^{\frac{1}{2}}} dx = \int \underbrace{(2x+1)}_{f(x)}^{-\frac{1}{2}} dx \\
 &= \frac{1}{2} \int \underbrace{2}_{f'(x)} (2x+1)^{-\frac{1}{2}} dx = \frac{1}{2} \left(\frac{1}{\frac{1}{2}} (2x+1)^{\frac{1}{2}} \right) + C \\
 &= \sqrt{2x+1} + C
 \end{aligned}$$

8. Exponenttifunktion integrooli

$$\begin{cases} D e^x = e^x \\ D e^{f(x)} = e^{f(x)} \cdot f'(x) \end{cases}$$

$$\begin{aligned} \int e^x dx &= e^x + C \\ \int f'(x) e^{f(x)} dx &= e^{f(x)} + C \end{aligned}$$

huom. Siisfunktion $f(x)$ derivaatto $f'(x)$ täytyy olla mukana integroitavassa funktiossa.

$$\text{Esim. a) } \int e^{3x} dx = \frac{1}{3} \int \underbrace{3}_{f'(x)} \underbrace{e^{3x}}_{f(x)} dx = \frac{1}{3} e^{3x} + C$$

$$\begin{aligned}
 \text{b) } \int (3x e^{x^2} + e^{-\frac{x}{2}}) dx &= \int 3x e^{x^2} dx + \int e^{-\frac{x}{2}} dx \\
 &= 3 \cdot \frac{1}{2} \int \underbrace{2x}_{f'(x)} \underbrace{e^{x^2}}_{f(x)} dx + (-2) \int \underbrace{-\frac{1}{2}}_{f'(x)} \underbrace{e^{-\frac{x}{2}}}_{f(x)} dx \\
 &= \frac{3}{2} e^{x^2} - 2 e^{-\frac{x}{2}} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \int (e^x + e^{-2x})^2 dx &= \int ((e^x)^2 + 2e^x e^{-2x} + (e^{-2x})^2) dx \\
 &= \int (e^{2x} + 2e^{-x} + e^{-4x}) dx \\
 &= \frac{1}{2} \int 2e^{2x} dx + 2(-1) \int -e^{-x} dx + \frac{1}{4} \int -4e^{-4x} dx \\
 &= \frac{1}{2} e^{2x} - 2e^{-x} - \frac{1}{4} e^{-4x} + C
 \end{aligned}$$

$$\text{d) } \int e^{x^2} dx$$