

huom. Määrätty integraali lausutaan integraaliksi C suostun pois (edellisen esimerkin). Siten määrätty integraali lausutaan C jättä pois!

5, 6, 9, 12

5.6 $A = \int_3^5 (5 - \frac{1}{2}x) dx = \left[5x - \frac{1}{2} \cdot \frac{1}{2} x^2 \right]_3^5$
 $= (5 \cdot 5 - \frac{1}{4} \cdot 5^2) - (5 \cdot 3 - \frac{1}{4} \cdot 3^2) = \dots = \underline{6}$

$$\begin{aligned} 5.9 \quad \int_a^4 (6x^2 - 8x) dx &= \left[6 \cdot \frac{1}{3} x^3 - 8 \cdot \frac{1}{2} x^2 \right]_a^4 = \left[2x^3 - 4x^2 \right]_a^4 \\ &= \underbrace{(2 \cdot 4^3 - 4 \cdot 4^2)}_{64} - (2a^3 - 4a^2) = 64 - 2a^3 + 4a^2 = 64 \end{aligned}$$

$$\Rightarrow -2a^3 + 4a^2 = 0 \quad \Rightarrow 2a^2(-a + 2) = 0$$

$$\Leftrightarrow a^2 = 0 \vee 4 \quad -a + 2 = 0 \quad \Leftrightarrow \underline{a = 0 \vee a = 2}$$

5.12 a) $\int_1^2 \frac{3x-x^4}{x} dx = \int_1^2 \frac{x(3-x^3)}{x} dx = \int_1^2 (3-x^3) dx = \left[3x - \frac{1}{4}x^4 \right]_1^2$
 $= (3 \cdot 2 - \frac{1}{4} \cdot 2^4) - (3 \cdot 1 - \frac{1}{4} \cdot 1^4) = \dots = -\frac{3}{4}$

$$\begin{aligned} b) \int_{-3}^{25} \frac{x^2 - 81}{x+9} dx &= \int_{-3}^{25} \frac{(x-9)(x+9)}{x+9} dx = \int_{-3}^{25} (x-9) dx = \left[\frac{1}{2}x^2 - 9x \right]_{-3}^{25} \\ &= \left(\frac{1}{2} \cdot 25^2 - 9 \cdot 25 \right) - \left(\frac{1}{2} \cdot (-3)^2 - 9 \cdot (-3) \right) = 56 \end{aligned}$$

$$\sqrt{x^2 - 81} = 0 \quad (\Rightarrow) x^2 = 81 \quad | \sqrt{\quad} \quad (\Rightarrow) x = \pm 9$$

5.19 $q(x) = |x-1|$

a) $f(x) = |x-1| = 0 \Leftrightarrow x-1 = 0 \Leftrightarrow x = 1$

b)  $f(x) = |x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1) = -x+1, & x < 1 \end{cases}$

$$c) \int_0^2 |x-1| dx = \int_0^1 |x-1| dx + \int_1^2 |x-1| dx$$

$$= \int_0^1 (-x+1) dx + \int_1^2 (x-1) dx = \dots = 1$$