

$$b) \quad \varphi(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + Cx + D$$

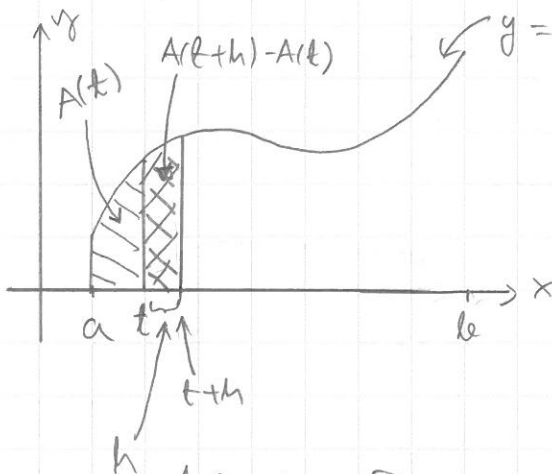
$$\varphi(0) = 2 \quad \Rightarrow \quad D = 2$$

$$\varphi'(x) = x^2 + x + C \quad \Rightarrow \quad \varphi'(1) = 1^2 + 1 + C = 2 + C = 0 \quad \Rightarrow \quad C = -2$$

$$\underline{\text{Vast.}} \quad \varphi(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 2$$

5. Analyyisin perusteita

Oletetaan $\varphi(x) \geq 0$ kun $x \in [a, b]$ ja $F'(x) = \varphi(x)$



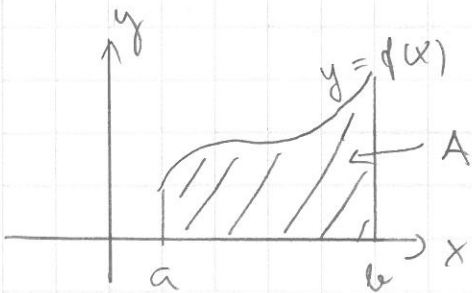
$$\varphi'(a) = \lim_{x \rightarrow a} \frac{\varphi(x) - \varphi(a)}{x - a} \stackrel{x-a=h}{=} \lim_{h \rightarrow 0} \frac{\varphi(a+h) - \varphi(a)}{h}$$

$$A'(t) = \lim_{h \rightarrow 0} \frac{A(t+h) - A(t)}{h} = \varphi(t)$$

$$\Rightarrow A(t) = F(t) + C$$

$$A(a) = F(a) + C = 0 \quad \Rightarrow \quad C = -F(a)$$

$$\text{Siis } A(t) = F(t) - F(a) = \int_a^t \varphi(x) dx \stackrel{\text{merk.}}{=} \left. F(x) \right|_a^t$$

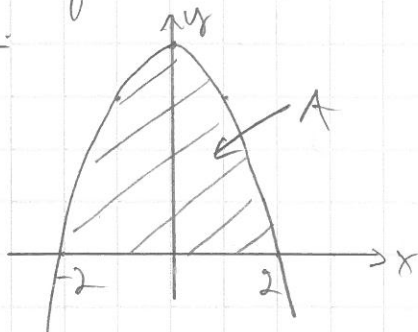


$$A = \int_a^b \varphi(x) dx = \left. F(x) \right|_a^b = F(b) - F(a)$$

"rajatut a:sta b:hen $F(x)$ "

Esim. Määritä käyrän $y = -x^2 + 4$ ja x -akselin välisen alueen pinta-ala.

Ratk.



$$0\text{-kohdat: } y = -x^2 + 4 = 0 \\ \Rightarrow 4 = x^2 \quad | \sqrt{\quad} \quad \Rightarrow \quad x = \pm 2$$

$$A = \int_{-2}^2 (-x^2 + 4) dx = \left. -\frac{1}{3}x^3 + 4x + C \right|_{-2}^2 \\ = \left(-\frac{1}{3} \cdot 2^3 + 4 \cdot 2 + C \right) - \left(-\frac{1}{3} \cdot (-2)^3 + 4 \cdot (-2) + C \right) \\ = \frac{32}{3} \quad (\approx 10,667)$$