

Merz. $\int f(x) dx = F(x) + C \quad \Leftrightarrow \quad F'(x) = f(x)$

Exm. Ouzo $f(x) = x + \sin x \cdot \cos x$ funktion $g(x) = 2\cos^2 x$ integralifunktio?

Ratk. Ouzo $f'(x) = g(x)$?

$$\begin{aligned} f'(x) &= 1 + \cos x \cdot \cos x + \sin x \cdot (-\sin x) \\ &= 1 + \cos^2 x - \sin^2 x \\ &= (\sin^2 x + \cos^2 x) + \cos^2 x - \sin^2 x \\ &= 2\cos^2 x = g(x) \Rightarrow \text{on} \end{aligned}$$

$$D \frac{f}{g} = \frac{f' \cdot g - g' \cdot f}{g^2}$$

$$D(f \cdot g) = f' \cdot g + f \cdot g'$$

$$D \sin x = \cos x$$

$$D \cos x = -\sin x$$

4. Polynomifunktion integrointi

Exm. $\int x^3 dx = \frac{1}{4} x^4 + C$

$$(D(\frac{1}{4}x^4 + C) = \frac{1}{4} \cdot 4x^3 = x^3)$$

Integroimisääntöjä

1. $\int k f(x) dx = k \int f(x) dx$, k vakio

2. $\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$

3. $\int k dx = kx + C$, k vakio

4. $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$, n vakio, $n \neq -1$ ($D(\frac{1}{n+1} x^{n+1} + C) = \frac{1}{n+1} \cdot \frac{x^n}{(n+1)} = x^n$ ✓)

4.4 a) $\int (1+2x)^2 dx = \int (1^2 + 2 \cdot 1 \cdot 2x + (2x)^2) dx$
 $= \int (1 + 4x + 4x^2) dx = x + 4 \cdot \frac{1}{2} x^2 + 4 \cdot \frac{1}{3} x^3 + C$
 $= x + 2x^2 + \frac{4}{3} x^3 + C$

b) $\int \frac{(2x-4)^2}{2} dx = \int \frac{(2x)^2 - 2 \cdot 2x \cdot 4 + 4^2}{2} dx = \int \frac{4x^2 - 16x + 16}{2} dx$
 $= \int (2x^2 - 8x + 8) dx = 2 \cdot \frac{1}{3} x^3 - 8 \cdot \frac{1}{2} x^2 + 8x + C$
 $= \frac{2}{3} x^3 - 4x^2 + 8x + C$

4.10 a) $f''(x) = 2x+1$

$$f'(x) = \int f''(x) dx = \int (2x+1) dx = 2 \cdot \frac{1}{2} x^2 + x + C = x^2 + x + C$$

$$f(x) = \int f'(x) dx = \int (x^2 + x + C) dx = \frac{1}{3} x^3 + \frac{1}{2} x^2 + Cx + D$$