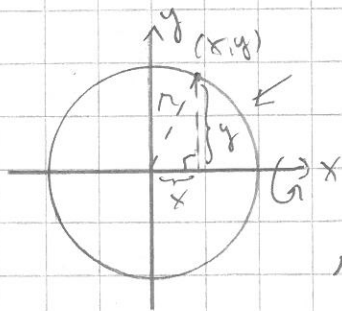


Esim. Pallon tilavuus ja pinta-ala



Pythagoras:  $x^2 + y^2 = r^2$

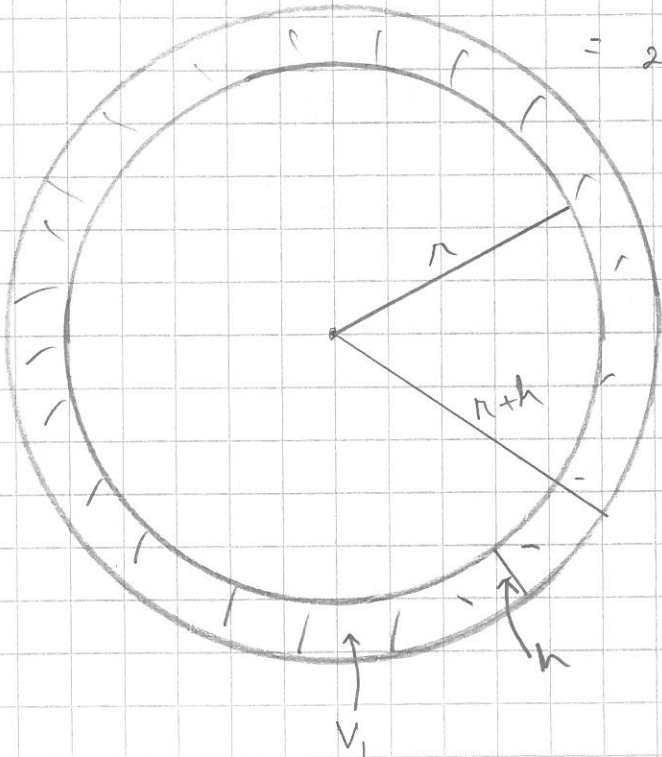
$r_p = (a, b)$ , säde:  $r$   
 $(x-a)^2 + (y-b)^2 = r^2$

symmetria -n

$$V = \pi \int_{-n}^n y^2 dx = \pi \int_{-n}^n (r^2 - x^2) dx$$

$$= 2\pi \int_0^n (r^2 - x^2) dx = 2\pi \left[ r^2 x - \frac{1}{3} x^3 \right]_0^n$$

$$= 2\pi \cdot \left( n^3 - \frac{1}{3} n^3 \right) = 2\pi \cdot \frac{2}{3} n^3 = \underline{\underline{\frac{4}{3} \pi n^3}}$$



$V(r) = \frac{4}{3} \pi r^3$

$V'_1(r) = \lim_{h \rightarrow 0} \frac{V(r+h) - V(r)}{h} = A(r)$

$\uparrow$   
 n-säteisen pallon  
 pinta-ala

Siksi  $\underline{A(r)} = V'_1(r) = \frac{d(\frac{4}{3} \pi r^3)}{dr} = \frac{4}{3} \pi \cdot 3r^2 = \underline{\underline{4\pi r^2}}$