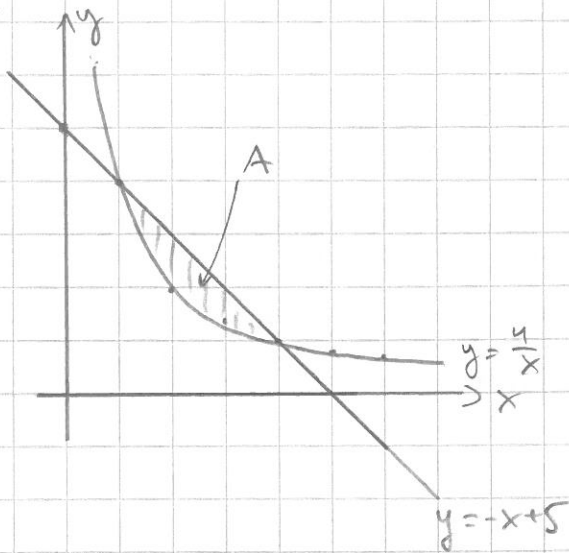


Exim. Laske käyrien  $x+y=5$  ja  $xy=4$  rajoittama alue.

Patk.



$$\begin{cases} x+y=5 & \Leftrightarrow y = -x+5 \\ xy=4 & | :x \neq 0 \Leftrightarrow y = \frac{4}{x} \end{cases}$$

$$\Rightarrow -x+5 = \frac{4}{x} \quad | \cdot x$$

$$\Leftrightarrow -x^2 + 5x = 4$$

$$\Leftrightarrow x^2 - 5x + 4 = 0 \quad \Leftrightarrow x = \begin{cases} 1 \\ 4 \end{cases}$$

$$A = \int_1^4 \left( (-x+5) - \frac{4}{x} \right) dx$$

$$= \int_1^4 \left( -\frac{1}{2}x^2 + 5x - 4 \ln|x| \right) dx$$

$$= \left( -\frac{1}{2} \cdot 4^2 + 5 \cdot 4 - 4 \ln 4 \right) - \left( -\frac{1}{2} \cdot 1^2 + 5 \cdot 1 - 4 \ln 1 \right)$$

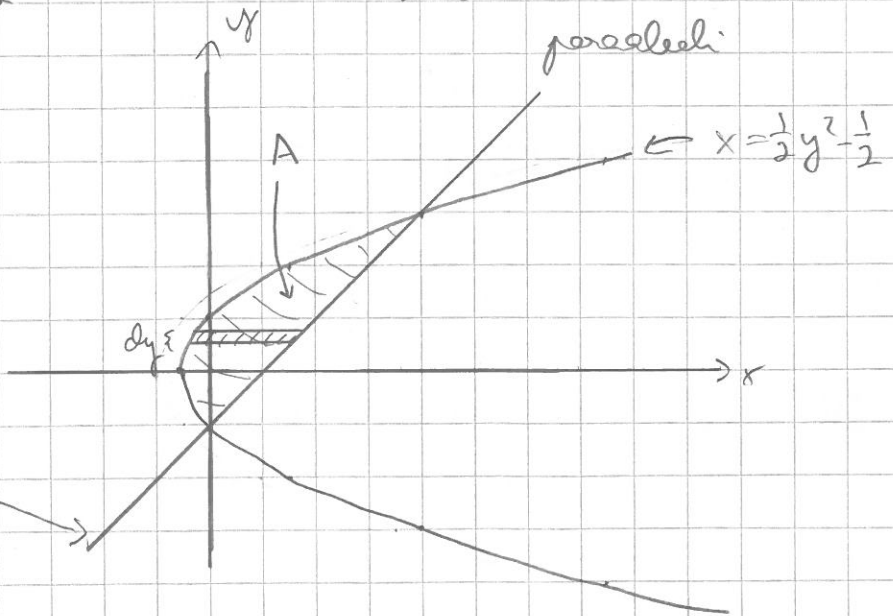
$$= \underline{\underline{\frac{15}{2} - 4 \ln 4}} \quad (\approx 1,95)$$

13.13

$$y^2 = 2x + 1 \quad \Leftrightarrow 2x = y^2 - 1 \quad | :2 \quad \Leftrightarrow x = \frac{1}{2}y^2 - \frac{1}{2}$$

y	$x = \frac{1}{2}y^2 - \frac{1}{2}$
0	$-\frac{1}{2}$
$\pm 1$	0
$\pm 2$	$\frac{3}{2}$
$\pm 3$	4
$\pm 4$	$7\frac{1}{2}$

$a = \frac{1}{2} > 0 \Rightarrow$  oikealle aukeava paraabeli



$$x - y - 1 = 0$$

$$\Leftrightarrow y = x - 1$$

Leikkaukkoalat:

$$\begin{cases} y^2 = 2x + 1 & \Leftrightarrow x = \frac{1}{2}y^2 - \frac{1}{2} \quad (=x_1) \\ x - y - 1 = 0 & \Leftrightarrow x = y + 1 \quad (=x_2) \end{cases}$$

$$\Rightarrow \frac{1}{2}y^2 - \frac{1}{2} = y + 1 \quad | \cdot 2 \quad \Leftrightarrow y^2 - 2y - 3 = 0 \quad \Leftrightarrow y = \begin{cases} -1 \\ 3 \end{cases}$$

$$A = \int_{-1}^3 (x_2 - x_1) dy = \int_{-1}^3 \left( (y+1) - \left( \frac{1}{2}y^2 - \frac{1}{2} \right) \right) dy = \int_{-1}^3 \left( -\frac{1}{2}y^2 + y + \frac{3}{2} \right) dy$$

$$= \int_{-1}^3 \left( -\frac{1}{2} \cdot \frac{1}{3}y^3 + \frac{1}{2}y^2 + \frac{3}{2}y \right) dy = \left( -\frac{1}{6} \cdot 3^3 + \frac{1}{2} \cdot 3^2 + \frac{3}{2} \cdot 3 \right) - \left( -\frac{1}{6} \cdot (-1)^3 + \frac{1}{2} \cdot (-1)^2 + \frac{3}{2} \cdot (-1) \right)$$

$$= \underline{\underline{\frac{16}{3}}}$$