

Exim,
$$\int \frac{e^{3x} + e^x}{e^x} dx = \int \frac{e^{3x}}{e^x} + \frac{e^x}{e^x} dx = \int (e^{2x} + 1) dx$$

$$= \int e^{2x} dx + \int 1 dx = \frac{1}{2} \int 2e^{2x} dx + x$$

$$= \frac{1}{2} e^{2x} + x + C$$

10.17 a)
$$\int \frac{\overbrace{\cos x}^{f'(x)}}{\underbrace{\sin x}_{g(x)}} dx = \ln|\sin x| + C = \ln(\sin x) + C, 0 < x < \frac{\pi}{2}$$
> 0 Rem $0 < x < \frac{\pi}{2}$

b)
$$\int \frac{\sin x}{4 \cos x} dx = \frac{1}{4} \int \frac{\sin x}{\cos x} dx = -\frac{1}{4} \int \frac{\overbrace{-\cos x}^{f'(x)}}{\underbrace{\cos x}_{g(x)}} dx = -\frac{1}{4} \ln|\cos x| + C$$

$$= -\frac{1}{4} \ln(\cos x) + C$$
> 0

c)
$$\int \frac{2 \cos x}{\sin^2 x} dx = 2 \int \frac{\overbrace{\cos x}^{f'(x)}}{\underbrace{(\sin x)^2}_{g(x)}} dx = 2 \cdot \frac{1}{-1} (\sin x)^{-1} + C$$

$$= -\frac{2}{\sin x} + C$$

10.18 a)
$$\int_1^e \frac{x-1}{x} dx = \int_1^e \left(\frac{x}{x} - \frac{1}{x} \right) dx = \int_1^e \left(1 - \frac{1}{x} \right) dx = \int_1^e x - \ln|x|$$

$$= (e - \underbrace{\ln e}_{=1}) - (1 - \underbrace{\ln 1}_{=0}) = \underline{e - 2}$$

b)
$$\int_0^1 \frac{\overbrace{e^x}^{f'(x)}}{\underbrace{e^x + 2}_{g(x)}} dx = \int_0^1 \ln|e^x + 2| = \ln(e^1 + 2) - \ln(\underbrace{e^0}_{=1} + 2)$$

$$= \ln(e + 2) - \ln 3$$

c)
$$\int_0^1 \frac{e^x + 2}{e^x} dx = \int_0^1 \left(\frac{e^x}{e^x} + \frac{2}{e^x} \right) dx = \int_0^1 (1 + 2e^{-x}) dx$$

$$= \int_0^1 (x - 2e^{-x}) = (1 - 2e^{-1}) - (0 - 2\underbrace{e^0}_{=1})$$

$$= 3 - 2e^{-1}$$

10.19 a)
$$\int \frac{1}{x \ln x} dx = \int \frac{1}{x} \cdot \frac{1}{\ln x} dx = \int \frac{\overbrace{1}^{f'(x)}}{\underbrace{\ln x}_{g(x)}} dx = \ln|\ln x| + C$$
> 0 Rem $x > 1$

b)
$$\int \frac{\ln x}{x} dx = \int \frac{1}{x} \cdot \underbrace{(\ln x)'}_{f'(x)} dx = \frac{1}{2} (\ln x)^2 + C = \underline{\ln(\ln x) + C}$$