

$$= 2 \int \sin x \, dx + \frac{1}{3} \int 3 \cos 3x \, dx + (-1) \int -e^{-x} \, dx$$

$$= 2(-\cos x) + \frac{1}{3} \sin 3x - e^{-x} + C$$

$$F(0) = -2 \underbrace{\cos 0}_{=1} + \frac{1}{3} \underbrace{\sin 0}_{=0} - \underbrace{e^0}_{=1} + C = -2 - 1 + C = 6 \Rightarrow C = 9$$

$$\Rightarrow \underline{F(x) = -2 \cos x + \frac{1}{3} \sin 3x - e^{-x} + 9}$$

$$3.10 \quad f''(x) = \cos x + 2 \sin 4x, \quad f'(0) = \frac{1}{2}, \quad f(0) = 4$$

$$f'(x) = \int (\cos x + 2 \sin 4x) \, dx$$

$$= \int \cos x \, dx + \int 2 \sin 4x \, dx$$

$$= \sin x + 2 \cdot \frac{1}{4} \int 4 \sin 4x \, dx = \sin x + \frac{1}{2} (-\cos 4x) + C$$

$$f'(0) = \underbrace{\sin 0}_{=0} - \frac{1}{2} \underbrace{\cos 0}_{=1} + C = -\frac{1}{2} + C = \frac{1}{2} \Rightarrow C = 1$$

$$f(x) = \int f'(x) \, dx = \int (\sin x - \frac{1}{2} \cos 4x + 1) \, dx$$

$$= \int \sin x \, dx - \frac{1}{2} \int \cos 4x \, dx + \int 1 \, dx$$

$$= -\cos x - \frac{1}{2} \cdot \frac{1}{4} \int 4 \cos 4x \, dx + x$$

$$= -\cos x - \frac{1}{8} \sin 4x + x + D$$

$$f(0) = -\underbrace{\cos 0}_{=1} - \frac{1}{8} \underbrace{\sin 0}_{=0} + 0 + D = -1 + D = 4 \Rightarrow D = 5$$

$$\underline{\text{Varl. } f(x) = -\cos x - \frac{1}{8} \sin 4x + x + 5}$$

$$3.15 \quad a) \int e^{\cos x} \sin x \, dx = - \int \underbrace{\sin x}_{f'(x)} \cdot \underbrace{e^{\cos x}}_{f(x)} \, dx = -e^{\cos x} + C$$

$$b) \int e^{3x} \cos e^{3x} \, dx = \frac{1}{3} \int \underbrace{3e^{3x}}_{f'(x)} \cos \underbrace{e^{3x}}_{f(x)} \, dx \quad D e^{3x} = e^{3x} \cdot 3$$

$$= \frac{1}{3} \sin e^{3x} + C$$

$$3.5 \quad f(x) = \cos x \sin x$$

$$F(x) = \int \cos x \sin x \, dx = \int \underbrace{\cos x}_{g'(x)} \underbrace{(\sin x)'}_{g(x)} \, dx = \frac{1}{2} (\sin x)^2 + C$$

$$F(\pi) = \frac{1}{2} \underbrace{(\sin \pi)_{=0}}^2 + C = C = 3 \quad \underline{\text{Varl. } F(x) = \frac{1}{2} \sin^2 x + 3}$$

$$\boxed{\text{TAI}} \quad F(x) = \int \cos x \sin x \, dx = \int \frac{1}{2} \cdot \underbrace{2 \sin x \cos x}_{=\sin 2x} \, dx = \frac{1}{2} \cdot \frac{1}{2} \int 2 \sin 2x \, dx = \frac{1}{4} (-\cos 2x) + C$$