8. Eksponenttifunktion integrointi

$$\begin{cases} De^x = e^x \\ De^{f(x)} = e^{f(x)} \cdot f'(x) \end{cases} \Rightarrow \boxed{\begin{aligned} \int e^x\,dx &= e^x + C \\ \int f'(x) e^{f(x)}\,dx &= e^{f(x)} + C \end{aligned}}$$

Huom. Sisäfunktion derivaatta $f'(x)$ täytyy olla integroitavassa funktiossa.

Esim. a) $\int e^{3x}\,dx = \frac{1}{3}\int \underbrace{3}_{f'(x)} e^{\overbrace{3x}^{=f(x)}}\,dx = \frac{1}{3}e^{3x} + C$

b) $\int (3x e^{x^2} + e^{-\frac{x}{2}})\,dx = \int 3x e^{x^2}\,dx + \int e^{-\frac{x}{2}}\,dx$ $\quad -\frac{x}{2} = -\frac{1}{2}x$

$= 3\cdot\frac{1}{2}\int \underbrace{2x}_{f'(x)} e^{\overset{f(x)}{x^2}}\,dx + (-2)\int -\frac{1}{2}e^{-\frac{x}{2}}\,dx = \frac{3}{2}e^{x^2} - 2e^{-\frac{x}{2}} + C$

c) $\int (e^x + e^{-2x})^2\,dx = \int ((e^x)^2 + 2e^x\cdot e^{-2x} + (e^{-2x})^2)\,dx$

$= \int e^{2x}\,dx + \int 2e^{-x}\,dx + \int e^{-4x}\,dx$

$= \frac{1}{2}\int 2e^{2x}\,dx + 2\cdot(-1)\int -e^{-x}\,dx + \frac{1}{-4}\int -4e^{-4x}\,dx$

$= \frac{1}{2}e^{2x} - 2e^{-x} - \frac{1}{4}e^{-4x} + C$

TAI: $\int (e^{2x} + 2e^{-x} + e^{-4x})\,dx \overset{\text{yritä}}{=} e^{2x} + 2e^{-x} + e^{-4x} + C$

Tark. $D(e^{2x} + 2e^{-x} + e^{-4x} + C) = e^{2x}\cdot 2 + 2e^{-x}\cdot(-1) + e^{-4x}\cdot(-4)$

2: pitää saada pois $\to$ kerroin: $\frac{1}{2}$

(−1): pois $\to$ kerroin: -1

(−4): pois $\to$ kerroin: $-\frac{1}{4}$

$\Rightarrow$ tulos: $\underline{\frac{1}{2}e^{2x} - 2e^{-x} - \frac{1}{4}e^{-4x} + C}$

d) $\int e^{x^2}\,dx$ $\quad\Rightarrow$ ei voi integroida

ei saada $Dx^2 = 2x$

8.13 $f(x) = 3 - e^{-x} = 0 \Leftrightarrow 3 = e^{-x} \quad | \ln \Leftrightarrow \ln 3 = \ln e^{-x}$

$\Leftrightarrow \ln 3 = -x \Leftrightarrow x = -\ln 3 \ (\approx -1{,}10)$

$A = \int_{-\ln 3}^{0} (3 - e^{-x})\,dx = \int_{-\ln 3}^{0} 3\,dx + \int_{-\ln 3}^{0} -e^{-x}\,dx = \Big/_{-\ln 3}^{0} 3x + \Big/_{-\ln 3}^{0} e^{-x}$

$= \left[3\cdot 0 - 3(-\ln 3)\right] + \left[\underbrace{e^0}_{=1} - \underbrace{e^{-(-\ln 3)}}_{e^{\ln 3} = 3}\right] = \underline{3\ln 3 - 2} \ (\approx 1{,}30)$