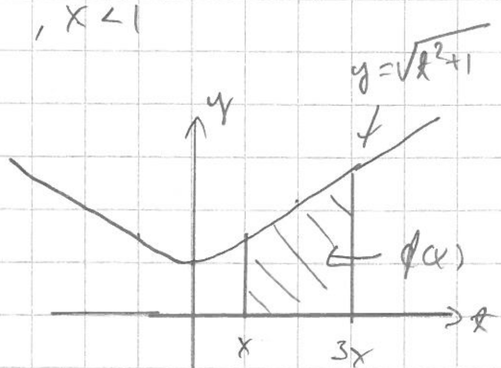
$$= \frac{1}{\frac{5}{2}}\underbrace{(x-1)^{\frac{5}{2}}}_{(x-1)^{2+\frac{1}{2}}} + C = \frac{2}{5}(x-1)^2\sqrt{x-1} + C$$

b) $$\int \frac{1-x}{\sqrt{1-x}}\,dx = \int (1-x)^1 \cdot (1-x)^{-\frac{1}{2}}\,dx = \int (1-x)^{\frac{1}{2}}\,dx$$

$$= -\int \underbrace{-1}_{f'(x)} \cdot (\underbrace{1-x}_{f(x)})^{\frac{1}{2}}\,dx = -\frac{1}{\frac{3}{2}}\underbrace{(1-x)^{\frac{3}{2}}}_{(1-x)^{1+\frac{1}{2}}} + C$$

$$= -\frac{2}{3}(1-x)\sqrt{1-x} + C \qquad , x < 1$$

7.20 $f: \underbrace{\mathbb{R}}_{x\in\mathbb{R}} \to \mathbb{R}$, $f(x) = \int_x^{3x} \sqrt{t^2+1}\,dt$



$$f(x) = \int_x^{3x} \underbrace{\sqrt{t^2+1}}_{g(t)}\,dt = \int_x^{3x} (t^2+1)^{\frac{1}{2}}\,dt$$

ei sisäda $D(t^2+1) = 2t$ $\Rightarrow$ ei voida integroida

$g(t)$, g jatkuva $\mathbb{R}$:ssä $\Rightarrow$ on olemassa integraalifunktio G

$$= \Big/_x^{3x} G(t) = G(3x) - G(x)$$

$D(3x)$ ei sisäfunktion derivaatta

$$f'(x) = D\big(G(3x) - G(x)\big) = G'(3x)\cdot 3 - G'(x)$$

$$= 3g(3x) - g(x) = \underline{3\sqrt{(3x)^2+1} - \sqrt{x^2+1}}$$

Ääriarvokohdat: $f'(x) = 3\sqrt{9x^2+1} - \sqrt{x^2+1} = 0$

$\Leftrightarrow 3\sqrt{9x^2+1} = \sqrt{x^2+1}$ $\quad | ()^2$ mol. puol. ≥ 0

$\Leftrightarrow 9(9x^2+1) = x^2+1$

$\Leftrightarrow 81x^2 + 9 = x^2 + 1 \quad \Leftrightarrow 80\underbrace{x^2}_{\geq 0} = -8$ $\lightning$ ei ratk.

f':llä ei 0-kohtia $\Rightarrow$ f:llä <u>ei ääriarvoja</u>

7.16 $f(x) = (2x-1)(x^2-x)^2$

$$F(x) = \int \underbrace{(2x-1)}_{g'(x)}\underbrace{(x^2-x)^2}_{g(x)}\,dx = \frac{1}{3}(x^2-x)^3 + C$$, F jatk. ja deriv.

$F'(x) = f(x) = (2x-1)(x^2-x)^2 = 0 \quad \Leftrightarrow x = \frac{1}{2}$ tai $x = 0$ tai $x = 1$

		0		$\frac{1}{2}$		1	
F'	$-$		$-$		$+$		$+$
F	$\searrow$		$\searrow$	min	$\nearrow$		$\nearrow$

pienin arvo: $F(\frac{1}{2}) = \frac{1}{3}\left((\frac{1}{2})^2 - \frac{1}{2}\right)^3 + C = -1$

$\Leftrightarrow C = -\frac{191}{192}$

<u>Vast. $F(x) = \frac{1}{3}(x^2-x)^3 - \frac{191}{192}$</u>