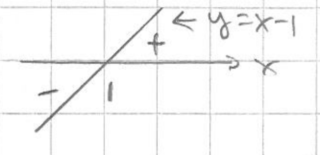
$$=\left(3\cdot 2-\tfrac{1}{4}\cdot 2^4\right)-\left(3\cdot 1-\tfrac{1}{4}\cdot 1^4\right)=(6-4)-\left(3-\tfrac{1}{4}\right)=\underline{-\tfrac{3}{4}}$$

b) $\displaystyle\int_{-3}^{25}\frac{x^2-\underset{9^2}{81}}{x+9}\,dx=\int_{-3}^{25}\frac{(x-9)\cancel{(x+9)}}{\cancel{x+9}}\,dx=\int_{-3}^{25}(x-9)\,dx=\Big/_{\!-3}^{25}\ \tfrac{1}{2}x^2-9x$

$$=\left(\tfrac{1}{2}\cdot 25^2-9\cdot 25\right)-\left(\tfrac{1}{2}\cdot(-3)^2-9\cdot(-3)\right)=56$$

5.19 $f(x)=|x-1|$

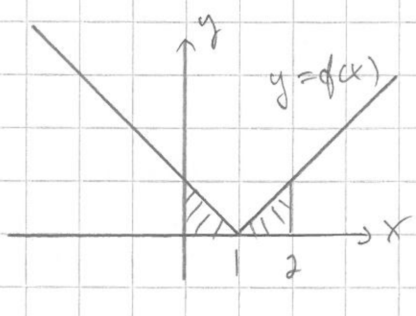
a) $f(x)=|x-1|=0 \iff x-1=0 \iff \underline{x=1}$

b) 

Testipisteet $x=0:\ 0-1=-1<0$

$x=2:\ 2-1=1>0$

$$\Rightarrow f(x)=\begin{cases}-(x-1)=-x+1, & x<1\\ x-1, & x\geq 1\end{cases}$$



c) $\displaystyle\int_0^2|x-1|\,dx=\int_0^1(-x+1)\,dx+\int_1^2(x-1)\,dx$

$\displaystyle=\Big/_{\!0}^{1}-\tfrac{1}{2}x^2+x\ +\Big/_{\!1}^{2}\ \tfrac{1}{2}x^2-x$

$=\left[\left(-\tfrac{1}{2}\cdot 1^2+1\right)-0\right]+\left[\left(\tfrac{1}{2}\cdot 2^2-2\right)-\left(\tfrac{1}{2}\cdot 1^2-1\right)\right]=\tfrac{1}{2}+\tfrac{1}{2}=\underline{1}$

5.16 a) $\displaystyle\int_0^2\left(\int_1^3(2x+t)\,dx\right)dt=\int_0^2\left(\Big/_{\!1}^{3}\underbrace{2\cdot\tfrac{1}{2}}_{=1}x^2+tx\right)dt$

$\displaystyle=\int_0^2\Big(\left(3^2+t\cdot 3\right)-\left(1^2+t\cdot 1\right)\Big)\,dt=\int_0^2(2t+8)\,dt$

$\displaystyle=\Big/_{\!0}^{2}\ \cancel{2}\cdot\tfrac{1}{\cancel{2}}t^2+8t\ =\left(2^2+8\cdot 2\right)-0=\underline{20}$

6. Potenssifunktion integrointi

Esim. $\displaystyle\int\sqrt{x}\,dx=\int x^{\frac{1}{2}}\,dx=\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1}+C=\frac{1}{\frac{3}{2}}\underbrace{x^{\frac{3}{2}}}_{x^1\cdot x^{\frac{1}{2}}}+C=\frac{2}{3}x\sqrt{x}+C$

6.3 a) $\displaystyle\int-3\sqrt{x}\,dx=-3\int x^{\frac{1}{2}}\,dx=-3\left(\frac{1}{\frac{3}{2}}\underbrace{x^{\frac{3}{2}}}_{x^1\cdot x^{\frac{1}{2}}}\right)+C=-2x\sqrt{x}+C,\ x\geq 0$

b) $\displaystyle\int\frac{4}{\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}}}\,dx=4\int x^{-\frac{1}{2}}\,dx=4\cdot\left(\frac{1}{\frac{1}{2}}x^{\frac{1}{2}}\right)+C=8\sqrt{x}+C,\ x>0$

c) $\displaystyle\int\frac{1}{x\sqrt{x}}\,dx=\int\frac{1}{x^1\cdot x^{\frac{1}{2}}}\,dx=\int\frac{1}{x^{\frac{3}{2}}}\,dx=\int x^{-\frac{3}{2}}\,dx=\frac{1}{-\frac{1}{2}}x^{-\frac{1}{2}}+C=-2\cdot\frac{1}{\sqrt{x}}+C$, $x>0$