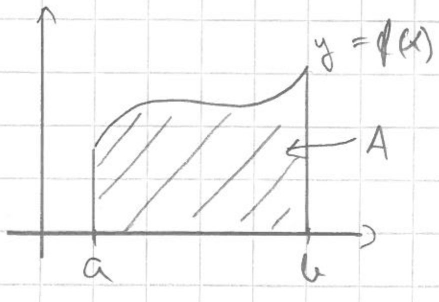



yläraja

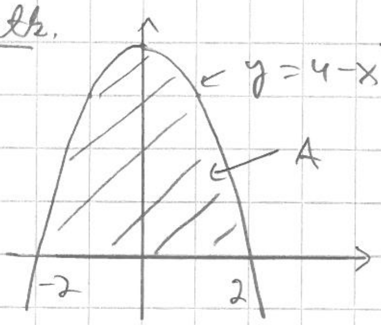
$$A = \int_a^b f(x)\,dx = \Big/_{a}^{b} F(x) = F(b) - F(a)$$

alaraja

"sijoitus a:sta b:hen F(x)"

Esim. Laske käyrän $y = 4 - x^2$ ja x-akselin rajaama pinta-ala.

Ratk.




0-kohdat: $y = 4 - x^2 = 0$

$\Leftrightarrow 4 = x^2 \ |\sqrt{\ } \Leftrightarrow x = \pm 2$

$$A = \int_{-2}^{2} (4 - x^2)\,dx = \Big/_{-2}^{2} 4x - \frac{1}{3}x^3 + C$$

$$= \left(4 \cdot 2 - \frac{1}{3} \cdot 2^3 + C\right) - \left(4 \cdot (-2) - \frac{1}{3} \cdot (-2)^3 + C\right)$$

$$= 8 - \frac{8}{3} + \cancel{C} + 8 - \frac{8}{3} - \cancel{C}$$

$$= {}^{3)}16 - \frac{16}{3} = \frac{32}{3} = 10\frac{2}{3} \quad (\approx 10{,}66667)$$

Huom. Edellisessä tehtävässä C supistui pois. Yleisesti määrättyä integraalia laskettaessa integroimisvakio C supistuu pois, joten C voidaan jättää pois!

5.6 $f(x) = 5 - \frac{1}{2}x$

$$A = \int_3^5 \left(5 - \frac{1}{2}x\right) dx = \Big/_{3}^{5} 5x - \frac{1}{2} \cdot \frac{1}{2}x^2$$

$$= \left(5 \cdot 5 - \frac{1}{4} \cdot 5^2\right) - \left(5 \cdot 3 - \frac{1}{4} \cdot 3^2\right) = 25 - \frac{25}{4} - 15 + \frac{9}{4} = \underline{6}$$

5.9 $$\int_a^4 (6x^2 - 8x)\,dx = \Big/_{a}^{4} 6 \cdot \frac{1}{3}x^3 - 8 \cdot \frac{1}{2}x^2 = \Big/_{a}^{4} 2x^3 - 4x^2$$

$$= \left(2 \cdot 4^3 - 4 \cdot 4^2\right) - \left(2 \cdot a^3 - 4 \cdot a^2\right) = 64 - 2a^3 + 4a^2 = 64$$

$\Leftrightarrow -2a^3 + 4a^2 = 0 \quad \Leftrightarrow -2a^2(a - 2) = 0 \quad \Leftrightarrow a^2 = 0 \ |\sqrt{\ }$ tai $a - 2 = 0$

$\Leftrightarrow \underline{a = 0 \text{ tai } a = 2}$

5.12 a) $$\int_1^2 \frac{3x - x^4}{x}\,dx = \int_1^2 \frac{\cancel{x}(3 - x^3)}{\cancel{x}}\,dx = \int_1^2 (3 - x^3)\,dx = \Big/_{1}^{2} 3x - \frac{1}{4}x^4$$