2. $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

3. $\int k\,dx = kx + C$

4. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$, $n \neq -1$

4.4 a) $\int (1+2x)^2\,dx = \int (1^2 + 2\cdot 1\cdot 2x + (2x)^2)\,dx = \int (1+4x+4x^2)\,dx$

$= x + 4\cdot\frac{1}{2}x^2 + 4\cdot\frac{1}{3}x^3 + C = x + 2x^2 + \frac{4}{3}x^3 + C$

b) $\int \frac{(2x-4)^2}{2}\,dx = \int \frac{(2x)^2 - 2\cdot 2x\cdot 4 + 4^2}{2}\,dx = \int \frac{4x^2 - 16x + 16}{2}\,dx$

$= \int (2x^2 - 8x + 8)\,dx = 2\cdot\frac{1}{3}x^3 - 8\cdot\frac{1}{2}x^2 + 8x + C$

$= \frac{2}{3}x^3 - 4x^2 + 8x + C$

DERIVOINTI ON HELPPOA, INTEGROINTI HANKALAA

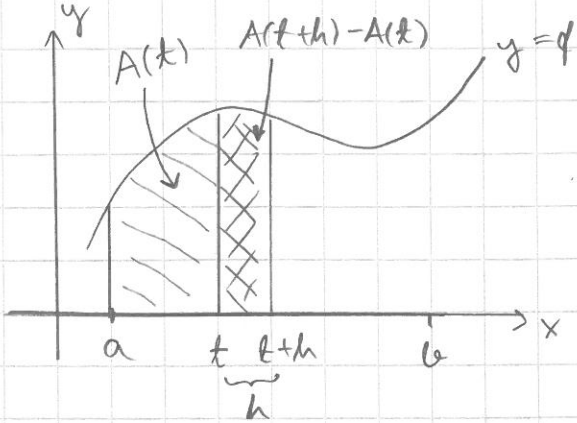
⇒ TARKISTAKAA INTEGROINNIT DERIVOIMALLA !

4.17 a) $\int \frac{x^2-9}{x+3}\,dx = \int \frac{x^2-3^2}{x+3}\,dx = \int \frac{(x-3)(x+3)}{x+3}\,dx$

$= \int (x-3)\,dx = \frac{1}{2}x^2 - 3x + C$

5. Analyysin peruslause

Olkoon $f(x) \geq 0$ kun $x \in [a,b]$ ja $F'(x) = f(x)$



$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$A'(t) = \lim_{h\to 0} \frac{A(t+h)-A(t)}{h} = f(t)$

$\Rightarrow A(t) = F(t) + C$

$A(a) = F(a) + C = 0 \iff C = -F(a)$

Siis $A(t) = F(t) - F(a) = \int_a^t f(x)\,dx = \Big/_a^t F(x)$