$$F(\pi)=3 \Leftrightarrow \frac{1}{2}(\underbrace{\sin\pi}_{=0})^2 + C = 3 \Leftrightarrow C=3$$

Vast. $F(x)=\frac{1}{2}\sin^2 x + 3$

TAI: $F(x)=\int \cos x \sin x \, dx = \int (\cos x)' \sin x \, dx$

$$= -\int \underbrace{-\sin x}_{g'(x)} \underbrace{(\cos x)}_{g(x)}' \, dx = -\frac{1}{2}(\cos x)^2 + C$$

TAI: $F(x)=\int \cos x \sin x \, dx = \frac{1}{2}\int \underbrace{2\sin x \cos x}_{=\sin 2x} \, dx$

$$= \frac{1}{2}\int \sin 2x \, dx$$

$$= \frac{1}{2}\cdot\frac{1}{2}\int 2\sin 2x \, dx = \frac{1}{4}(-\cos 2x) + C$$

10. Osamäärän integrointi

$$D\ln|x| = \begin{cases} D\ln x = \frac{1}{x} & , x>0 \\ D\ln(-x) = \frac{1}{-x}\cdot(-1) = \frac{1}{x} & , x<0 \end{cases} \Rightarrow \boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$$

$$D\ln|f(x)| = \begin{cases} D\ln f(x) = \frac{1}{f(x)}\cdot f'(x) = \frac{f'(x)}{f(x)} & , f(x)>0 \\ D\ln(-f(x)) = \frac{1}{-f(x)}\cdot(-f'(x)) = \frac{f'(x)}{f(x)} & , f(x)<0 \end{cases}$$

$$\Rightarrow \boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C}$$

10.7 a) $\int \frac{dx}{2x-6} = \int \frac{1}{2x-6}\,dx = \frac{1}{2}\int \frac{\overbrace{2}^{f'(x)}}{\underbrace{2x-6}_{f(x)}}\,dx = \frac{1}{2}\ln|\underbrace{2x-6}_{>0 \text{ kun } x>3}| + C$

$$= \frac{1}{2}\ln(2x-6) + C, \quad x>3$$

b) $\int \frac{dx}{(2x-6)^3} = \int \frac{1}{(2x-6)^3}\,dx = \int (2x-6)^{-3}\,dx = \frac{1}{2}\int \underbrace{2}_{f'(x)}\underbrace{(2x-6)}_{f(x)}^{-3}\,dx$

$$= \frac{1}{2}\cdot\frac{1}{-2}(2x-6)^{-2} + C = -\frac{1}{4(2x-6)^2} + C, \quad x>3$$

c) $\int \frac{dx}{\sqrt{2x-6}} = \int \frac{1}{\sqrt{2x-6}}\,dx = \int (2x-6)^{-\frac{1}{2}}\,dx = \frac{1}{2}\int 2(2x-6)^{-\frac{1}{2}}\,dx$

$$= \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}(2x-6)^{\frac{1}{2}} + C = \sqrt{2x-6}, \quad x>3$$