21.4

a) $D\,e^{3x} = e^{3x}\cdot 3 = 3e^{3x}$

b) $D\,e^{-x} = e^{-x}\cdot(-1) = -e^{-x}$

c) $D\,2^{3x} = 2^{3x}\ln 2\cdot 3$

d) $D\,0{,}14^{2x} = 0{,}14^{2x}\ln 0{,}14\cdot 2$

21.7

$f(x) = e^{2x} - 2e^{x} + e^{-x}$

a) $f'(x) = e^{2x}\cdot 2 - 2e^{x} + e^{-x}\cdot(-1) = 2e^{2x} - 2e^{x} - e^{-x}$

b) $f'(0) = 2e^{0} - 2e^{0} - e^{0} = 2\cdot 1 - 2\cdot 1 - 1 = \underline{-1}$

21.8

a) $f(x) = 2x - e^{3x}$

$f'(x) = 2 - e^{3x}\cdot 3 = 0 \iff e^{3x} = \frac{2}{3} \quad |\ln$

$\iff 3x = \ln\frac{2}{3} \iff \underline{x = \frac{\ln\frac{2}{3}}{3}}$

b) $f(x) = x^2 e^{2x}$

$f'(x) = 2x e^{2x} + x^2 e^{2x}\cdot 2 = 2x e^{2x}(1+x) = 0$

$\iff x = 0$ tai $\underbrace{e^{2x}}_{>0} = 0$ ↯ tai $1 + x = 0 \iff \underline{x = 0 \text{ tai } x = -1}$

21.9

a) $\ln 8 - \ln 2 + \frac{1}{2}\ln 9 = \ln\frac{8}{2} + \ln 9^{\frac{1}{2}} = \ln 4 + \ln\sqrt{9}$

$= \ln(4\cdot 3) = \underline{\ln 12}$

b) $\ln e^{2} - 2\overbrace{\ln 1}^{=0} = 2\ln e = 2\cdot 1 = \underline{2}$

c) $\ln\frac{e^3}{2} = \ln e^3 - \ln 2 = \underline{3 - \ln 2}$

21.15

a) $f(x) = x^2 e^{-x}$

$f'(x) = 2x e^{-x} + x^2 e^{-x}\cdot(-1) = x e^{-x}(2 - x) = 0$

$\iff x = 0$ tai $\underbrace{e^{-x}}_{>0} = 0$ tai $2 - x = 0 \iff \underline{x = 0 \text{ tai } x = 2}$

b) $f(x) = \frac{e^{-x}}{x^2}, \quad x \neq 0$

$f'(x) = \frac{e^{-x}\cdot(-1)\cdot x^2 - e^{-x}\cdot 2x}{(x^2)^2} = 0 \quad |\cdot x^4 \neq 0$

$\iff -x\underbrace{e^{-x}}_{>0}(x + 2) = 0 \iff \underline{x = 0 \text{ tai } x = -2}$

22.1

a) $D(3\ln x) = 3\cdot\frac{1}{x} = \frac{3}{x}$

b) $D\ln 4x = \frac{1}{4x}\cdot 4 = \frac{1}{x}$ $\quad$ [TAI: $D\ln 4x = D(\ln 4 + \ln x) = \frac{1}{x}$]

c) $D\ln x^3 = \frac{1}{x^3}\cdot 3x^2 = \frac{3}{x}$ $\quad$ [TAI: $D\ln x^3 = D(3\ln x) = 3\cdot\frac{1}{x} = \frac{3}{x}$]

22.3

$f(x) = \ln(1 - x^2)$

a) $1 - x^2 > 0$; vast. yhtälö $1 - x^2 = 0 \iff x^2 = 1 \;\sqrt{}\iff x = \pm 1$

$\underline{-1 < x < 1}$

b) $f'(x) = \frac{1}{1 - x^2}\cdot(-2x) = \underline{-\frac{2x}{1 - x^2}}$

c) $f'(x) = -\frac{2x}{1 - x^2} = 0 \quad |\cdot(1 - x^2) \neq 0 \iff -2x = 0 \iff \underline{x = 0}$

22.4

a) $D\log_9 x = \underline{\frac{1}{x\ln 9}}, \quad x > 0$

b) $D\lg x = D\log_{10} x = \underline{\frac{1}{x\ln 10}}, \quad x > 0$

c) $D\lg 5x = D\log_{10} 5x = \frac{1}{5x\ln 10}\cdot 5 = \underline{\frac{1}{x\ln 10}}, \quad x > 0$

22.6

a) $f(x) = x\ln x, \quad x > 0$

$f'(x) = 1\cdot\ln x + x\cdot\frac{1}{x} = \ln x + 1 = 0 \iff \ln x = -1 \quad |e^{()}$

$\iff \underline{x = e^{-1} = \frac{1}{e}}$

b) $f(x) = \frac{\ln x}{x}, \quad x > 0$

$f'(x) = \frac{\frac{1}{x}\cdot x - \ln x\cdot 1}{x^2} = \frac{1 - \ln x}{x^2} = 0 \quad |\cdot x^2 \neq 0$

$\iff 1 - \ln x = 0 \iff \ln x = 1 \quad |e^{()} \iff \underline{x = e^1 = e}$

22.14

a) $f(x) = x\ln x + x, \quad x > 0$

$f'(x) = 1\cdot\ln x + x\cdot\frac{1}{x} + 1 = \ln x + 2 = 0$

$\iff \ln x = -2 \quad |e^{()} \iff e^{\ln x} = e^{-2} \iff \underline{x = e^{-2} = \frac{1}{e^2}}$

b) $f(x) = \frac{\ln x}{x^2}, \quad x > 0$

$f'(x) = \frac{\frac{1}{x}\cdot x^2 - \ln x\cdot 2x}{(x^2)^2} = \frac{x - 2x\ln x}{x^4} = 0 \quad |\cdot x^4 \neq 0$

$\iff x - 2x\ln x = x(1 - 2\ln x) = 0 \iff x = 0$ ↯ tai $1 - 2\ln x = 0$

$\iff \ln x = \frac{1}{2} \quad |e^{()} \iff e^{\ln x} = e^{\frac{1}{2}} \iff \underline{x = e^{\frac{1}{2}} = \sqrt{e}}$

22.17

a) $f(x) = 3\ln x - \ln 2x = 3\ln x - (\ln 2 + \ln x) = \underline{2\ln x - \ln 2}$

$f'(x) = 2\cdot\frac{1}{x} = \underline{\frac{2}{x}}, \quad x > 0$

b) $f(x) = \ln x^2 + \ln\frac{2}{x} = 2\ln x + (\ln 2 - \ln x) = \underline{\ln x + \ln 2}$

$f'(x) = \underline{\frac{1}{x}}, \quad x > 0$

c) $f(x) = \ln\sqrt{x} - \ln\sqrt[3]{x} = \ln x^{\frac{1}{2}} - \ln x^{\frac{1}{3}} = \frac{1}{2}\ln x - \frac{1}{3}\ln x$

$f'(x) = \frac{1}{6}\cdot\frac{1}{x} = \underline{\frac{1}{6x}}, \quad x > 0$ $\qquad = \underline{\frac{1}{6}\ln x}$

23.1

$f(x) = e^{x^2 - 4x}$, f jatk. ja derivoi. $\mathbb{R}$:ssä

$f'(x) = e^{x^2 - 4x}\cdot(2x - 4) = 0 \quad |: e^{x^2 - 4x} > 0 \iff 2x - 4 = 0 \iff x = 2$

		2	
f'	$-$		$+$
f	↘		↗

$f'(0) = -4 < 0 \qquad f(3) = 2e^{-3} \approx 0{,}10 > 0$

<u>minimiarvo: $f(2) = e^{-4}$</u>

23.3

$f(x) = \frac{\ln 2x}{x}$, f jatk. ja derivoi. väl. $[1, 4]$

$f'(x) = \frac{\frac{1}{2x}\cdot 2\cdot x - \ln 2x\cdot 1}{x^2} = \frac{1 - \ln 2x}{x^2} = 0 \quad |\cdot x^2 \neq 0$

$\iff 1 - \ln 2x = 0 \iff \ln 2x = 1 \quad |e^{()} \iff 2x = e^1 \iff x = \frac{e}{2}$

– päätepisteet: $f(1) = \frac{\ln 2}{1} = \ln 2 \approx 0{,}69$

$f(4) = \frac{\ln(2\cdot 4)}{4} = \frac{\ln 8}{4} = \frac{\ln 2^3}{4} = \frac{3\ln 2}{4} \approx 0{,}52$

– f':n 0-kohdat: $f\left(\frac{e}{2}\right) = \frac{\ln\left(2\cdot\frac{e}{2}\right)}{\frac{e}{2}} = \frac{\ln e}{\frac{e}{2}} = \frac{1}{\frac{e}{2}} = \frac{2}{e} \approx 0{,}74$

$\Rightarrow$ <u>suurin arvo: $\frac{2}{e}$, pienin arvo: $\frac{\ln 8}{4} = \frac{3\ln 2}{4}$</u>

23.4

$f(x) = x^2 e^{x}$, f jatk. ja derivoi. väl. $[-4, 1]$

$f'(x) = 2x e^{x} + x^2\cdot e^{x} = x e^{x}(2 + x) = 0$

$\iff x = 0$ tai $\underbrace{e^{x}}_{>0} = 0$ ↯ tai $2 + x = 0 \iff x = 0$ tai $x = -2$

– päätepisteet: $f(-4) = 16e^{-4} \approx 0{,}29$, $f(1) = e \approx 2{,}72$

– f':n 0-kohdat: $f(0) = 0$, $f(-2) = 4e^{-2} \approx 0{,}54$

$\Rightarrow$ <u>suurin arvo: e, pienin arvo: 0</u>

23.5

$f(x) = e^{\frac{1}{3}x^3 + x^2 + x}$, f jatk. ja derivoi. $\mathbb{R}$:ssä

$f'(x) = \underbrace{e^{\frac{1}{3}x^3 + x^2 + x}}_{>0}\cdot\underbrace{(x^2 + 2x + 1)}_{(x+1)^2 \geq 0} \geq 0$ aina ja

$f'(x) = 0 \iff x = -1$ (kerrannaisjuurta) $\Rightarrow$ f aidosti kasvava $\mathbb{R}$:ssä

23.9

$f(x) = \ln x + \ln(12 - x)$ $\qquad \left.\begin{array}{l} x > 0 \\ 12 - x > 0 \iff x < 12 \end{array}\right\} \Rightarrow 0 < x < 12$

f jatk. ja derivoi. väl. $]0, 12[$

$f'(x) = \frac{1}{x} + \frac{1}{12 - x}\cdot(-1) = 0 \quad |\cdot x(12 - x)$

$\iff 12 - x - x = 0 \iff 2x = 12 \iff x = 6$

		0		6		12
f'		$+$			$-$	
f		↗			↘	

$f'(1) = \frac{10}{11} > 0 \qquad f'(10) = -\frac{2}{5} < 0$

<u>f kasvava välillä $]0, 6]$</u>

23.14

$f(x) = \ln x + \ln(x^2 + x)$, f jatk. ja derivoi. kun $x > 0$

$f'(x) = \frac{1}{x} + \frac{1}{x^2 + x}\cdot(2x + 1) = 0 \quad |\cdot x(x + 1) \neq 0$

$\iff (x + 1) + (2x + 1) = 0 \iff 3x = -2 \iff x = -\frac{2}{3}$ ↯

$f'(1) = 1 + \frac{3}{2} = \frac{5}{2} > 0 \Rightarrow f'(x) > 0$ aina kun $x > 0$

$\Rightarrow$ <u>f aidosti kasvava kun $x > 0$</u>