11.14 $g(2)=5$, $g'(2)=-3$, $s(x)=x^3 g(x)$

$s'(x)=3x^2 g(x)+x^3 g'(x)$

$s'(2)=3\cdot 2^2 g(2)+2^3 g'(2)=3\cdot 4\cdot 5+8\cdot(-3)=60-24=\underline{36}$

12.1 $f(x)=\dfrac{21}{\underbrace{x^4}_{\geq 0}+7}$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x)=\dfrac{0\cdot(x^4+7)-21\cdot 4x^3}{(x+2)^2}=\dfrac{-84x^3}{(x+2)^2}=0 \quad |\cdot(x+2)^2$

$\Leftrightarrow -84x^3=0 \Leftrightarrow x^3=0 \quad |\sqrt[3]{\ } \Leftrightarrow x=0$

		0	
f'	+		−
f	↗		↘

$f(-1)=\frac{28}{3}$, $f'(1)=-\frac{28}{3}$

a) f kasvava väl. $\underline{]-\infty,0]}$ ja vähenevä väl. $\underline{[0,\infty[}$

b) maksimikohta: $\underline{x=0}$, maksimiarvo: $f(0)=\frac{21}{7}=\underline{3}$

12.3 $f(x)=\dfrac{1-x^2}{1+\underbrace{x^2}_{\geq 0}}$, f jatk. ja deriv. väl. $[-2,3]$

$f'(x)=\dfrac{-2x(1+x^2)-(1-x^2)\cdot 2x}{(1+x^2)^2}=\dfrac{-4x}{(1+x^2)^2}=0 \quad |\cdot(1+x^2)^2\neq 0$

$\Leftrightarrow -4x=0 \Leftrightarrow x=0$

f'	+		−	
f	↗		↘	
	−2	0		3

$f'(-1)=1>0$, $f'(1)=-1<0$

$f(-2)=-\frac{3}{5}$, $f(3)=\underline{-\frac{4}{5}}$ pienin arvo

$f(0)=\underline{1}$ suurin arvo

12.5 $f(x)=\dfrac{x^4}{8x-4x^2}$, $8x-4x^2=4x(2-x)=0 \Leftrightarrow x=\begin{cases}0\\2\end{cases}$

f jatk. ja deriv. kun $x\neq 0$ ja $x\neq 2$

$f'(x)=\dfrac{4x^3(8x-4x^2)-x^4(8-8x)}{(8x-4x^2)^2}=\dfrac{-8x^5+24x^4}{(8x-4x^2)^2}=0 \quad |\cdot()^2$

$\Leftrightarrow -8x^5+24x^4=0 \Leftrightarrow 8x^4(-x+3)=0 \Leftrightarrow x=\begin{cases}0 & \text{ei}\\3 & \text{kyllä}\end{cases}$

		0		2		3	
f'	+		+		+		−
f	↗		↗		↗		↘
						max	

$f'(-1)=\frac{2}{9}>0$, $f'(1)=1>0$

$f'(2{,}5)=6{,}25>0$, $f'(4)=-2<0$

maksimikohta: $\underline{x=3}$, maksimiarvo: $\underline{f(3)=-\frac{27}{4}}$

12.9 $h(x)=\dfrac{x^7}{\underbrace{x^2+1}_{\geq 0}}$, h jatk. ja deriv. $\mathbb{R}$:ssä

$h'(x)=\dfrac{7x^6(x^2+1)-x^7\cdot 2x}{(x^2+1)^2}=\dfrac{5x^8+7x^6}{(x^2+1)^2}\geq 0$ aina (≥ 0, ≥ 0)

ja $h'(x)=0 \Leftrightarrow x=0$ (terassikohta) $\Rightarrow h$ on aidosti kasvava $\mathbb{R}$:ssä $\Rightarrow$ ei ääriarvoja

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12.18 $h(x)=\dfrac{x^2-4}{\underbrace{x^2+2}_{\geq 0}}+3$, h jatk. ja deriv. $\mathbb{R}$:ssä

$h'(x)=\dfrac{2x(x^2+2)-(x^2-4)\cdot 2x}{(x^2+2)^2}=\dfrac{12x}{(x^2+2)^2}=0 \quad |\cdot()^2\neq 0$

$\Leftrightarrow 12x=0 \Leftrightarrow x=0$

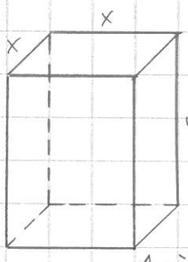
		0	
h'	−		+
h	↘		↗

$h'(-1)=-\frac{4}{3}<0$, $h'(1)=\frac{4}{3}>0$

pienin arvo: $h(0)=1$

$\Rightarrow h(x)>0$ aina $\Rightarrow$ väite

13.3



tilavuus $V=x^2y=1$ (dm³)

$\Leftrightarrow y=\frac{1}{x^2}$

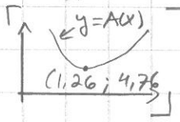
[rajatilanne: $y=\frac{1}{x^2}=0$ ei ratk.]

Pellin kulun:

$A(x)=x^2+4xy=x^2+4x\cdot\frac{1}{x^2}=x^2+\frac{4}{x}$

A jatk. ja deriv. kun $x>0$

$A'(x)=2x+\dfrac{0\cdot x-4\cdot 1}{x^2}=2x-\frac{4}{x^2}=0 \quad |\cdot x^2$



$\Leftrightarrow 2x^3-4=0 \Leftrightarrow x^3=2 \quad |\sqrt[3]{\ } \Leftrightarrow x=\sqrt[3]{2}\approx 1{,}2599$ (dm)

		$\sqrt[3]{2}$	
A'	−		+
A	↘		↗
	0	min	

$A'(1)=-2<0$ $\quad A'(2)=3>0$

$y=\frac{1}{(\sqrt[3]{2})^2}\approx 0{,}62996$

Vast. pohjaneliön sivu: 12,6 cm, korkeus: 6,3 cm

13.5 $f(t)=5-\dfrac{2-t}{t^2-6t+10}$, $t^2-6t+10=0$, $D=(-6)^2-4\cdot 1\cdot 10=-4<0$ ei ratk.

f jatk. ja deriv. väl. $[0,6]$

$f'(t)=-\dfrac{-1\cdot(t^2-6t+10)-(2-t)(2t-6)}{(t^2-6t+10)^2}=\dfrac{-t^2+4t-2}{(t^2-6t+10)^2}=0 \quad |\cdot()^2$

$\Leftrightarrow -t^2+4t-2=0 \Leftrightarrow t=2\pm\sqrt{2}\approx\begin{cases}0{,}5858\\3{,}4142\end{cases}$

- päätepisteet: $f(0)=4{,}8$, $f(6)=5{,}4$
- f':n 0-kohdat: $f(2-\sqrt{2})\approx 4{,}793$, $f(2+\sqrt{2})\approx 6{,}2071$ suurin arvo

Vast. Elukoita enintään 3,4 vuoden kuluttua korkeimmillaan alusta jollain niistä on n. 6200

13.6 $f(x)=x+\frac{1}{x}$, f jatk. ja deriv. kun $x>0$

$f'(x)=1+\dfrac{0\cdot 1-1\cdot 1}{x^2}=1-\frac{1}{x^2}=0 \quad |\cdot x^2$

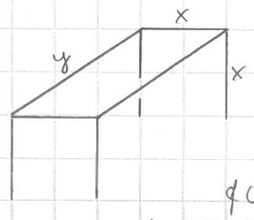
$\Leftrightarrow x^2-1=0 \Leftrightarrow x^2=1 \quad |\sqrt{\ } \Leftrightarrow x=\pm 1$

		0	
f'	−		+
f	↘		↗
		min	

$f'(\frac{1}{2})=-3<0$, $f'(2)=\frac{3}{4}>0$

pienin arvo: $f(1)=1+\frac{1}{1}=\underline{2}$

13.7



tilavuus: $x^2y=65 \Leftrightarrow y=\frac{65}{x^2}$

[rajatilanne: $y=\frac{65}{x^2}=0$ ei ratk.]

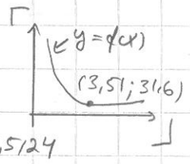
Pultkee kerreitaan:

$f(x)=6x+2y=6x+2\cdot\frac{65}{x^2}=6x+\frac{130}{x^2}$

f jatk. ja deriv. kun $x>0$

$f'(x)=6+\dfrac{0\cdot x^2-130\cdot 2x}{(x^2)^2}=6-\dfrac{260}{x^3}=0 \quad |\cdot x^3$

$\Leftrightarrow 6x^3-260=0 \Leftrightarrow x^3=\frac{260}{6} \quad |\sqrt[3]{\ } \Leftrightarrow x=\sqrt[3]{\frac{260}{6}}\approx 3{,}524$



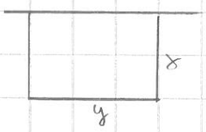
		$\sqrt[3]{\frac{260}{6}}$	
f'	−		+
f	↘		↗
	0	min	

$f'(1)=-254<0$ $\quad f'(4)\approx 1{,}94>0$

$y=\dfrac{65}{(\sqrt[3]{\frac{260}{6}})^2}\approx 5{,}2686$

Vast. päätypelion sivu: $x\approx\underline{3{,}5\text{ m}}$ ja pituus: $y\approx\underline{5{,}3\text{ m}}$

13.10



pinta-ala: $xy=3000 \Leftrightarrow y=\frac{3000}{x}$

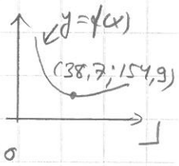
[rajatilanne: $y=\frac{3000}{x}=0$ ei ratk.]

aitaa kulun: $f(x)=2x+y=2x+\frac{3000}{x}$

f jatk. ja deriv. kun $x>0$

$f'(x)=2+\dfrac{0\cdot x-3000\cdot 1}{x^2}=2-\frac{3000}{x^2}=0 \quad |\cdot x^2$

$\Leftrightarrow 2x^2-3000=0 \Leftrightarrow x=\pm\sqrt{1500}\approx 38{,}7298$



		$\sqrt{1500}$	
f'	−		+
f	↘		↗
	0		

$f'(1)=-2998<0$, $f'(40)=\frac{1}{8}>0$

$y=\dfrac{3000}{\sqrt{1500}}\approx 77{,}4597$

Vast. rannan suuntainen sivu: $y\approx\underline{77{,}5\text{ m}}$

rantaa vastaan $\perp$ sivu: $x\approx\underline{38{,}7\text{ m}}$

14.1 $s(x)=5-3x$, $u(x)=\frac{1}{2-2x}$

a) $(u\circ s)(x)=u(s(x))=u(5-3x)=\dfrac{1}{2-2(5-3x)}$

$=\dfrac{1}{6x-8}$, $x\neq\frac{4}{3}$ $\quad (x\in\mathbb{R}\setminus\{\frac{4}{3}\})$

14.3 $h(x)=(u\circ s)(x)$

a) $h(x)=(4x-1)^6$, $s(x)=4x-1$, $u(x)=x^6$

b) $h(x)=\left(\frac{x}{x+1}\right)^3$, $s(x)=\frac{x}{x+1}$, $u(x)=x^3$