7.7 $y = -x^2+3$

b) $y(-1) = -(-1)^2+3 = 2 \Rightarrow$ piste : $(-1, 2)$

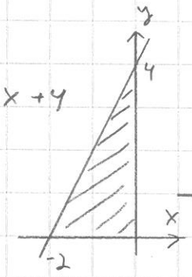
$y' = -2x \Rightarrow k_t = y'(-1) = -2\cdot(-1) = 2$

tangentti : $y - 2 = 2(x-(-1)) \Leftrightarrow y = 2x+4$

$x=0$: $y = 2\cdot 0+4 = 4$

$y=0$: $0 = 2x+4 \Leftrightarrow x = -2$

$A = \frac{1}{2}\cdot|-2|\cdot|4| = \frac{1}{2}\cdot 2\cdot 4 = \underline{4}$



7.9 Leikkauskohdat : $\begin{cases} y = x^2-5x+2 = f(x) \\ y = x^2-7x+8 = g(x) \end{cases}$

$\Rightarrow x^2-5x+2 = x^2-7x+8 \Leftrightarrow 2x = 6 \Leftrightarrow x = 3$

$\begin{cases} f'(x) = 2x-5 \Rightarrow k_{t_1} = f'(3) = 2\cdot 3-5 = 1 \\ g'(x) = 2x-7 \Rightarrow k_{t_2} = g'(3) = 2\cdot 3-7 = -1 \end{cases}$

$\Rightarrow k_{t_1}\cdot k_{t_2} = 1\cdot(-1) = -1$

$\Rightarrow$ leikkauspisteeseen piirretyt tangentit $\perp$ $\Rightarrow$ väite

7.16 Leikkauspiste : $\begin{cases} y = -\frac{1}{4}x^2+\frac{3}{2}x+\frac{3}{4} = f(x) \\ y = 2x^2+1 = g(x) \end{cases}$

$\Rightarrow -\frac{1}{4}x^2+\frac{3}{2}x+\frac{3}{4} = 2x^2+1 \quad |\cdot 4 \Leftrightarrow 9x^2-6x+1 = 0$

$\Leftrightarrow (3x-1)^2 = 0 \quad |\sqrt{\ } \Leftrightarrow 3x-1 = 0 \Leftrightarrow x = \frac{1}{3}$

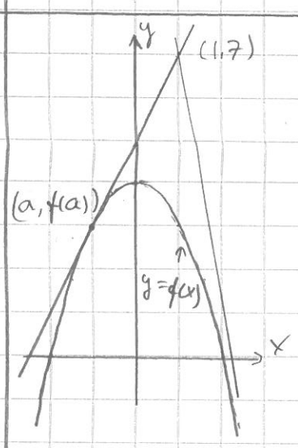
$f(\frac{1}{3}) = g(\frac{1}{3}) = \frac{11}{9} \Rightarrow$ vain 1 piste $(\frac{1}{3}, \frac{11}{9})$

$\begin{cases} f'(x) = -\frac{1}{2}x+\frac{3}{2} \Rightarrow k_{t_1} = f'(\frac{1}{3}) = \frac{4}{3} \\ g'(x) = 4x \Rightarrow k_{t_2} = g'(\frac{1}{3}) = \frac{4}{3} \end{cases}$

$\Rightarrow$ yhteiseen pisteeseen piirretyt tangentit samat

$\Rightarrow$ käyrät sivuavat toisiaan

7.20



$y = f(x) = -x^2+4$

Sivuamispiste : $(a, -a^2+4)$

$f'(x) = -2x \Rightarrow k_t = f'(a) = -2a$

tangentti : $y-(-a^2+4) = -2a(x-a)$

Piste $(1,7)$ on tangentilla :

$7+a^2-4 = -2a(1-a)$ sij.

$\Leftrightarrow a^2+3 = -2a+2a^2$

$\Leftrightarrow -a^2+2a+3 = 0 \Leftrightarrow a = \begin{cases} -1 \\ 3 \end{cases}$

$a=-1$: $\underline{y = 2x+5}$

$a = 3$: $\underline{y = -6x+13}$

TAI : $\begin{cases} k_t = f'(a) = -2a \\ k_t = \frac{\Delta y}{\Delta x} = \frac{(-a^2+4)-7}{a-1} = \frac{-a^2-3}{a-1} \end{cases}$

$\Rightarrow \frac{-a^2-3}{a-1} = -2a \quad |\cdot(a-1) \Leftrightarrow -a^2-3 = -2a(a-1)$

$\Leftrightarrow a^2-2a-3 = 0 \ \ldots$

8.1 $f(x) = x^3+3x^2+5$, f jatkuva ja derivoituva $\mathbb{R}$:ssä

$f'(x) = 3x^2+6x = 3x(x+2) = 0 \Leftrightarrow x = \begin{cases} 0 \\ -2 \end{cases}$

f'	+	−	+	
f	↗	↘	↗	
		-2	0	

f aidosti kasvava väl. $]-\infty, -2]$ ja $[0, \infty[$

f –"– vähenevä väl. $[-2, 0]$

8.3 $f(x) = -3x^4-4x^3+36x^2$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = -12x^3-12x^2+72x = -12x(x^2+x-6) = 0$

$\Leftrightarrow x = 0$ tai $x^2+x-6 = 0 \Leftrightarrow x = 0$ tai $x = \begin{cases} 2 \\ -3 \end{cases}$

f'	+	−	+	−	
f	↗	↘	↗	↘	
		-3 max	0 min	2 max	

$f'(-4) = 288 > 0$, $f'(-1) = -72 < 0$

$f'(1) = 48 > 0$, $f'(3) = -216 < 0$

maksimikohdat : $x = -3$ ja $x = 2$

minimikohta : $x = 0$

8.5 $f(x) = x^3-6x^2+5$, f jatk. ja deriv. kun $x \geq -1$

$f'(x) = 3x^2-12x = 3x(x-4) = 0 \Leftrightarrow x = \begin{cases} 0 \\ 4 \end{cases}$

f'	+	−	+	
f	↗	↘	↗	
	-1	0	4	

maksimiarvo : $f(0) = \underline{5}$

minimiarvot : $f(-1) = \underline{-2}$, $f(4) = \underline{-27}$

8.7 f : 4 $\quad g$: 1 $\quad h$: 3

8.12 $f(x) = 5x^5-27x^3-2$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = 25x^4-81x^2 = x^2(25x^2-81) = 0$

$\Leftrightarrow x = 0$ tai $x = \pm\sqrt{\frac{81}{25}} = \pm\frac{9}{5}$

f'	+	−	−	+	
f	↗	↘	↘	↗	
	$-\frac{9}{5}$ max	0	$\frac{9}{5}$ min		

$f'(\pm 2) = 76 > 0$, $f'(\pm 1) = -56 < 0$

maksimikohta : $\underline{x = -\frac{9}{5}}$

minimikohta : $\underline{x = \frac{9}{5}}$

terassikohta : $\underline{x = 0}$

8.17 b) $f(x) = 2x^3-15x^2+24x+a$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = 6x^2-30x+24 = 0 \Leftrightarrow x = \begin{cases} 1 \\ 4 \end{cases}$

f'	+	−	+	
f	↗	↘	↗	
	1 max	4 min		

maksimiarvo : $f(1) = 11+a = 8 \Leftrightarrow \underline{a = -3}$

minimi –"– : $f(4) = \ldots = \underline{-19}$

8.18 $f(x) = -x^4+8x^3+14x^2$, f jatk. ja deriv. $\mathbb{R}$:ssä

b) $f'(x) = -4x^3+24x^2+28x = 4x(-x^2+6x+7) = 0$

$\Leftrightarrow x = 0$ tai $-x^2+6x+7 = 0 \Leftrightarrow x = 0$ tai $x = \begin{cases} -1 \\ 7 \end{cases}$

f'	+	−	+	−	
f	↗	↘	↗	↘	
	-1 max	0 min	7 max		

$f'(-2) = 72 > 0$, $f'(-\frac{1}{2}) = -\frac{15}{2} < 0$

$f'(1) = 48 > 0$, $f'(8) = -288 < 0$

maksimiarvot : $f(-1) = \underline{5}$, $f(7) = \underline{1029}$

minimiarvo : $f(0) = \underline{0}$

9.1 $f(x) = x^3-6x^2+9x$, f jatk. ja deriv. väl. $[0, 3]$

$f'(x) = 3x^2-12x+9 = 0 \Leftrightarrow x = \begin{cases} 1 \\ 3 \end{cases}$

– pääte-kohdat : $f(0) = 0$, $f(3) = 0$

– f':n 0-kohdat : $f(1) = 4$

$\Rightarrow$ suurin arvo : $f(1) = 4$, pienin arvo : $f(0) = f(3) = 0$

9.3 $f(x) = 3x^4+4x^3-36x^2+10$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = 12x^3+12x^2-72x = 12x(x^2+x-6) = 0$

$\Leftrightarrow x = 0$ tai $x^2+x-6 = 0 \Leftrightarrow x = 0$ tai $x = \begin{cases} 2 \\ -3 \end{cases}$

f'	−	+	−	+	
f	↘	↗	↘	↗	
	-3 min	0 max	2 min		

$f'(-4) = -288 < 0$, $f'(-1) = 72 > 0$

$f'(1) = -48 < 0$, $f'(3) = 216 > 0$

$\left.\begin{array}{l} f(-3) = -179 \\ f(2) = -54 \end{array}\right\} \Rightarrow$ pienin arvo : $\underline{f(-3) = -179}$

$\left.\begin{array}{l} f(0) = 10 \\ \text{vrm. } f(3) = 37 \end{array}\right\}$ $\Rightarrow f(0)$ ei ole f:n suurin arvo

$\Rightarrow f$:llä ei ole suurinta arvoa

9.4 Väite $16x^3+30x^2 < 3x^4+900$, $x \in \mathbb{R}$

Tod. Väite $\Leftrightarrow \underbrace{-3x^4+16x^3+30x^2-900}_{=f(x)} < 0$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = -12x^3+48x^2+60x = 12x(-x^2+4x+5) = 0$

$\Leftrightarrow x = 0$ tai $-x^2+4x+5 = 0 \Leftrightarrow x = 0$ tai $x = \begin{cases} -1 \\ 5 \end{cases}$

f'	+	−	+	−	
f	↗	↘	↗	↘	
	-1 max	0	5 max		

$f'(-2) = 168 > 0$, $f'(-\frac{1}{2}) = -\frac{33}{2}$

$f'(1) = 96 > 0$, $f'(6) = -504 < 0$

$\left.\begin{array}{l} f(1) = -889 < 0 \\ f(5) = -25 < 0 \end{array}\right\} \Rightarrow$ suurin arvo : -25

$\Rightarrow f(x) < 0$ aina $\Rightarrow$ väite