4.14 a) $f(x) < 0 \Leftrightarrow -3 < x < -2$ tai $2 < x < 3$

b) $f'(x) < 0 \Leftrightarrow 0 < x < 3$ tai $x > 3$

c) $|f'(x)| = 1 \Leftrightarrow f'(x) = \pm 1 \Leftrightarrow x = \pm 4$ tai $x = \pm 2$

d) $|f'(x)| < 1 \Leftrightarrow -1 < f'(x) < 1 \Leftrightarrow x < -4$ tai $-2 < x < 2$ tai $x > 4$

5.1 $f(x) = 4x^2$

$$f'(3) = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{4x^2 - 4\cdot 3^2}{x-3} = \lim_{x\to 3} \frac{4(x^2-3^2)}{x-3}$$

$$= \lim_{x\to 3} \frac{4(x-3)(x+3)}{x-3} = \lim_{x\to 3} 4(x+3) = 4(3+3) = \underline{24}$$

TAI: $$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{4(3+h)^2 - 4\cdot 3^2}{h}$$

$$= \lim_{h\to 0} \frac{4((3+h)^2 - 3^2)}{h} = \lim_{h\to 0} \frac{4((9+6h+h^2)-9)}{h}$$

$$= \lim_{h\to 0} \frac{4(6h+h^2)}{h} = \lim_{h\to 0} \frac{4h(6+h)}{h} = \lim_{h\to 0} 4(6+h) = 4(6+0) = 24$$

5.3 $f(x) = \dfrac{2}{x^2-2}$

$$f'(4) = \lim_{x\to 4} \frac{f(x)-f(4)}{x-4} = \lim_{x\to 4} \frac{\frac{2}{x^2-2} - \frac{2}{4^2-2}}{x-4} \overset{7)}{=} \lim_{x\to 4} \frac{\frac{2}{x^2-2} - \frac{1}{7}}{x-4}$$

$$= \lim_{x\to 4} \frac{\frac{14-(x^2-2)}{7(x^2-2)}}{x-4} = \lim_{x\to 4} \frac{16-x^2}{7(x^2-2)(x-4)}$$

$$= \lim_{x\to 4} \frac{(4-x)(4+x)}{7(x^2-2)(-(4-x))} = \lim_{x\to 4} \frac{4+x}{-7(x^2-2)}$$

$$= \frac{4+4}{-7(4^2-2)} = \frac{8}{-7\cdot 14} = \underline{-\frac{4}{49}}$$

5.5 $f(x) = 4x^2 + 3$

a) $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(4(x+h)^2+3)-(4x^2+3)}{h}$$

$$= \lim_{h\to 0} \frac{4x^2+8xh+4h^2+3-4x^2-3}{h} = \lim_{h\to 0} \frac{8xh+4h^2}{h}$$

$$= \lim_{h\to 0} (8x+4h) = 8x + 4\cdot 0 = \underline{8x}$$

TAI: $$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{(4x^2+3)-(4a^2+3)}{x-a}$$

$$= \lim_{x\to a} \frac{4x^2-4a^2}{x-a} = \lim_{x\to a} \frac{4(x^2-a^2)}{x-a} = \lim_{x\to a} \frac{4(x-a)(x+a)}{x-a}$$

$$= \lim_{x\to a} 4(x+a) = 4(a+a) = 4\cdot 2a = 8a \Rightarrow f'(x) = \underline{8x}$$

b) $f'(-10) = 8\cdot(-10) = \underline{-80}$, $f'(1) = 8\cdot 1 = \underline{8}$

5.16 $f(x) = 2x^2 - x$

$$f'(3) = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{(2x^2-x)-(2\cdot 3^2-3)}{x-3} = \lim_{x\to 3} \frac{2x^2-x-15}{x-3}$$

$$\overset{*}{=} \lim_{x\to 3} \frac{2(x-3)(x+\frac{5}{2})}{x-3} = \lim_{x\to 3} 2(x+\tfrac{5}{2}) = 2\cdot(3+\tfrac{5}{2}) = \underline{11}$$

* $2x^2 - x - 15 = 0 \Leftrightarrow x = \begin{cases} 3 \\ -\frac{5}{2} \end{cases}$

5.18 $f(x) = 2x^2 - 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(2(x+h)^2-1)-(2x^2-1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)-2-2x^2+2}{h} = \lim_{h\to 0} \frac{4xh+2h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(4x+2h)}{h} = \lim_{h\to 0} (4x+2h) = 4x+2\cdot 0 = 4x$$

TAI: $$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{(2x^2-1)-(2a^2-1)}{x-a} = \lim_{x\to a} \frac{2(x^2-a^2)}{x-a}$$

$$= \lim_{x\to a} \frac{2(x-a)(x+a)}{x-a} = \lim_{x\to a} 2(x+a) = 2(a+a) = 4a \Rightarrow f'(x) = 4x$$

a) $f'(x) = 4x = 10 \Leftrightarrow \underline{x = \frac{5}{2}}$

b) $f'(x) = 4x = 0 \Leftrightarrow \underline{x = 0}$

6.1 a) $Dx^8 = 8x^7$ b) $D\,5x^3 = 15x^2$ c) $D(-2x^6) = -12x^5$

d) $D\frac{2x^{10}}{5} = \frac{2}{5}\cdot 10x^9 = 4x^9$ e) $D(-17x) = -17$ f) $D\,13 = 0$

6.2 a) $f(x) = 2x^5 - 12x^3 + 5x \Rightarrow f'(x) = 10x^4 - 36x^2 + 5$

b) $f(x) = 3x^4 + \frac{3}{4}x^2 + 5 \Rightarrow f'(x) = 12x^3 + \frac{3}{2}x$

6.7 a) $f(x) = 2x^3 - 5x + 4 \Rightarrow f'(x) = 6x^2 - 5 \Rightarrow \underline{4}$

b) $f(x) = 3x^2 - 5x + 4 \Rightarrow f'(x) = 6x - 5 \Rightarrow \underline{2}$

c) $f(x) = 2x^5 - 5x^2 + 4x \Rightarrow f'(x) = 10x^4 - 10x + 4 \Rightarrow \underline{5}$

6.14 $h(t) = 18t - 5t^2$

Nopeus on kuljetun matkan muutosnopeus eli derivaatta:

$v(t) = h'(t) = 18 - 10t \quad (\frac{m}{s})$

a) $v(1) = 18 - 10\cdot 1 = 8 > 0 \Rightarrow \underline{8{,}0\,\frac{m}{s}\ \text{ylöspäin}}$

$v(2) = 18 - 10\cdot 2 = -2 < 0 \Rightarrow \underline{2{,}0\,\frac{m}{s}\ \text{alaspäin}}$

b) $v(t) = 18 - 10t = 0 \Leftrightarrow t = 1{,}8 \Rightarrow \underline{1{,}8\,s}$

c) $h(1{,}8) = 18\cdot 1{,}8 - 5\cdot 1{,}8^2 = 16{,}2 \Rightarrow \underline{16\,m}$

6.15 $f(x) = x^2 + bx + c$

$f'(x) = 2x + b$

$\begin{cases} f(1) = 1^2 + b\cdot 1 + c = 1 + b + c = 2 \\ f'(1) = 2\cdot 1 + b = 2 + b = 5 \end{cases} \Leftrightarrow \underline{b = 3}$ sij.

$\Rightarrow 1 + 3 + c = 2 \Leftrightarrow \underline{c = -2}$

6.18 $f(x) = 5x^2 + 14x - 3$

a) $f'(x) = 10x + 14$ b) $f''(x) = 10$ c) $f^{(3)}(x) = 0$

7.1 $f(x) = x^2 - 3$

$f(1) = 1^2 - 3 = -2 \Rightarrow$ piste $(1, -2)$

a) $f'(x) = 2x \Rightarrow k_t = f'(1) = 2\cdot 1 = 2$

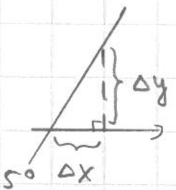
tangentti: $y - (-2) = 2(x-1) \Leftrightarrow \underline{y = 2x - 4}$

b) $k_t \cdot k_n = -1 \Leftrightarrow k_n = -\frac{1}{k_t} = -\frac{1}{2}$

normaali: $y - (-2) = -\frac{1}{2}(x-1) \Leftrightarrow \underline{y = -\frac{1}{2}x - \frac{3}{2}}$

7.3 $y = -0{,}16x^2 + 1{,}1x + 1$

$y' = -0{,}32x + 1{,}1$



a) $k_t = y'(2) = -0{,}32\cdot 2 + 1{,}1 = 0{,}46$

$\tan\alpha = \frac{\Delta y}{\Delta x} = k_t = 0{,}46 \Rightarrow \alpha = 24{,}70° \approx 25°$

b) $y = -0{,}16x^2 + 1{,}1x + 1 = 0 \Leftrightarrow x \approx \begin{cases} -0{,}81296 \\ 7{,}68796 \end{cases}$

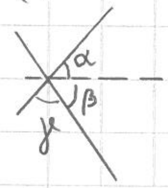
$k_t = y'(7{,}68796) \approx -1{,}36015$

$\tan\alpha = k_t \Rightarrow \alpha \approx -53{,}676° \Rightarrow$ n. $\underline{54°}$

7.4 Leikkauspisteet: $\begin{cases} y = x^2 - 2 \\ y = x^2 - 8x + 16 \end{cases}$

$\Rightarrow x^2 - 2 = x^2 - 8x + 16 \Leftrightarrow 8x = 18 \Leftrightarrow x = \frac{9}{4}$

$\begin{cases} y = x^2 - 2 \Rightarrow y' = 2x \Rightarrow k_{t_1} = y'(\frac{9}{4}) = \frac{9}{2} \\ y = x^2 - 8x + 16 \Rightarrow y' = 2x - 8 \Rightarrow k_{t_2} = y'(\frac{9}{4}) = -\frac{7}{2} \end{cases}$



$\tan\alpha = k_{t_1} = \frac{9}{2} \Rightarrow \alpha \approx 77{,}47°$

$\tan\beta = k_{t_2} = -\frac{7}{2} \Rightarrow \beta \approx -74{,}05°$

$\Rightarrow |\alpha - \beta| \approx |77{,}47° - (-74{,}05°)| = 151{,}526° > 90°$

$\Rightarrow \gamma = 180° - 151{,}526° = 28{,}474° \approx \underline{28°}$