24.1

myt : 120

1 v:n kulutus : $1{,}108 \cdot 120$

2 — " — : $1{,}108^2 \cdot 120$

⋮

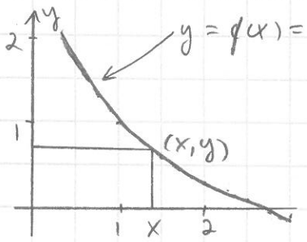
x — " — : $f(x) = 1{,}108^x \cdot 120$

a) $f(10) = 1{,}108^{10} \cdot 120 \approx 334{,}641 \approx \underline{330 \text{ elukaa}}$

b) $f'(x) = 1{,}108^x \ln 1{,}108 \cdot 120$ ($\frac{\text{kpl}}{\text{vuosi}}$)

$f'(10) \approx 34{,}3196 \approx \underline{34 \ \frac{\text{kpl}}{\text{vuosi}}}$

24.3



$y = f(x) = 1 - \ln x = 0 \iff \ln x = 1 \ | e^{()} \iff x = e$

Suorakulmion pinta-ala:

$A(x) = xy = x(1 - \ln x) = x - x\ln x$

A jatk. ja deriv. kun $x > 0$

$A'(x) = 1 - (1 \cdot \ln x + x \cdot \frac{1}{x})$

$= -\ln x = 0$

$\iff \ln x = 0 \ | e^{()} \iff x = e^0 = 1$

A'	+		−
A	↗		↘
	0		
		max	

$A'(\frac{1}{2}) = \ln 2 > 0$, $A'(2) = -\ln 2 < 0$

suurin arvo : $A(1) = 1 - 1\underbrace{\ln 1}_{=0} = \underline{1}$

24.7

a) $y(t) = y_0 e^{-kt}$

$y(30) = y_0 e^{-30k} = \frac{1}{2} y_0 \ | : y_0$

$\iff e^{-30k} = \frac{1}{2} \ | \ln \iff -30k = \ln \frac{1}{2} = \overbrace{\ln 1}^{=0} - \ln 2 = -\ln 2$

$\iff k = \frac{\ln 2}{30}$ ($\frac{1}{\text{vuosi}}$)

b) $y(t) = y_0 e^{-kt} = 0{,}1 y_0 \ | : y_0 \iff e^{-kt} = 0{,}1 \ | \ln$

$\iff -kt = \ln 0{,}1 \iff t = -\frac{\ln 0{,}1}{k} = -\frac{\ln 0{,}1}{\frac{\ln 2}{30}} \approx 99{,}6578 \approx 100$

$\Rightarrow$ v. $1986 + 100 = \underline{2086}$

c) $y'(t) = y_0 e^{-kt} \cdot (-k)$

$y'(40) = y_0 e^{-\frac{\ln 2}{30} \cdot 40} \cdot (-\frac{\ln 2}{30}) \approx -0{,}0092 y_0$ ($\frac{1}{\text{vuosi}}$)

$\Rightarrow$ vähenee nopeudella $\underline{0{,}0092 y_0 \ \frac{1}{\text{vuosi}}}$

24.10

alussa : 250

1 viikon kulutus : $x \cdot 250$

2 — " — : $x^2 \cdot 250$

3 — " — : $x^3 \cdot 250 = \frac{1}{2} \cdot 250 \ | : 250$

$\iff x^3 = \frac{1}{2} \ | \sqrt[3]{\ } \iff x = \sqrt[3]{\frac{1}{2}}$ ($\approx 0{,}793701$)

a) $f(t) = (\sqrt[3]{\frac{1}{2}})^t \cdot 250 \text{ g} = \underline{(\frac{1}{2})^{\frac{t}{3}} \cdot 250 \text{ g}}$

b) $f(t) = e^{\ln (\frac{1}{2})^{\frac{t}{3}}} \cdot 250 \text{ g} = \underline{e^{t \cdot \frac{1}{3} \ln \frac{1}{2}} \cdot 250 \text{ g}}$

c) $f'(t) = e^{t \cdot \frac{1}{3} \ln \frac{1}{2}} \cdot \frac{1}{3} \ln \frac{1}{2} \cdot 250 \ \frac{\text{g}}{\text{viikko}}$

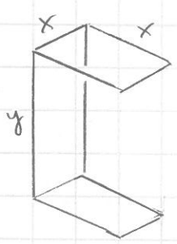
$f'(1{,}5) \approx -40{,}8441 \Rightarrow$ vähenee n. $\underline{41 \ \frac{\text{g}}{\text{viikko}}}$

K3.

a) $\lim_{x \to -1} \frac{x^2 + x}{3x + 3} \overset{(\frac{0}{0})}{=} \lim_{x \to -1} \frac{x(x+1)}{3(x+1)} = \lim_{x \to -1} \frac{x}{3} = \frac{-1}{3} = \underline{-\frac{1}{3}}$

b) $\lim_{x \to \frac{1}{2}} \frac{6x^3 - 3x^2}{2x - 1} \overset{(\frac{0}{0})}{=} \lim_{x \to \frac{1}{2}} \frac{3x^2(2x-1)}{2x-1} = \lim_{x \to \frac{1}{2}} 3x^2 = 3 \cdot (\frac{1}{2})^2 = \underline{\frac{3}{4}}$

K25.



rautalankaa : $8x + 2y = 200 \ | : 2$

$\iff y = 100 - 4x$

rajoituksia : $y = 0 \iff x = 25$

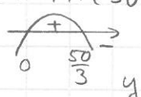
Varastimen tilavuus :

$V(x) = x^2 y = x^2(100 - 4x) = 100x^2 - 4x^3$

V jatk. ja deriv. väl. $[0, 25]$

$V'(x) = 200x - 12x^2 = 4x(50 - 3x) = 0 \iff x = 0$ tai $x = \frac{50}{3} \approx 16{,}67$

V'	+		−
V	↗		↘
	0		
		max	
		$\frac{50}{3}$	
			25



$y = 100 - 4 \cdot \frac{50}{3} \approx 33{,}333$

Vast. pohjasärmä : 16,7 cm, korkeus : $y \approx 33{,}3$ cm

K41.

$V = \pi x^2 y = 3 \text{ (dm}^3) \iff y = \frac{3}{\pi x^2}$

Pahvia kulun :

$A(x) = \pi x^2 + 2\pi x \cdot y = \pi x^2 + 2\pi x \cdot \frac{3}{\pi x^2}$

$= \pi x^2 + \frac{6}{x}$, A jatk. ja deriv. kun $x > 0$

$A'(x) = 2\pi x + \frac{0 \cdot x - 6 \cdot 1}{x^2} = 2\pi x - \frac{6}{x^2} = 0 \ | \cdot x^2$

$\iff 2\pi x^3 - 6 = 0 \iff x^3 = \frac{3}{\pi} \ | \sqrt[3]{\ } \iff x = \sqrt[3]{\frac{3}{\pi}} \approx 0{,}9847$ (dm)

A'	−		+
A	↘		↗
	0		
		$\sqrt[3]{\frac{3}{\pi}}$	

$A'(0{,}1) \approx -599 < 0$, $A'(1) \approx 0{,}28 > 0$

$y = \frac{3}{\pi (\sqrt[3]{\frac{3}{\pi}})^2} \approx 0{,}9847$ (dm)

Vast. pohjan säde = korkeus $\approx$ 9,8 cm