16. Funktion derivoitto

Kertausta:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ kpl}}$$

$$a^{-n} = \frac{1}{a^n}, \quad a \neq 0$$

$$a^{\frac{k}{n}} = \left(\sqrt[n]{a}\right)^k = \sqrt[n]{a^k}, \quad a > 0$$

$$D\,x^n = n\,x^{n-1}$$

$$D\,(f(x))^n = n\,(f(x))^{n-1} \cdot f'(x)$$

Esim. $D\sqrt{x} = D\,x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Yleisesti:

$$\boxed{\begin{aligned} &D\,x^r = r\,x^{r-1}, \quad r \in \mathbb{R},\ x > 0 \\ &D\,(f(x))^r = r\,(f(x))^{r-1} \cdot f'(x), \quad r \in \mathbb{R},\ f(x) > 0 \end{aligned}}$$

16.2 a) $f(x) = \sqrt[3]{x^2-5} = (x^2-5)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(x^2-5)^{\frac{1}{3}-1}\cdot 2x = \frac{1}{3}(x^2-5)^{-\frac{2}{3}}\cdot 2x = \frac{1}{3}\frac{1}{(x^2-5)^{\frac{2}{3}}}\cdot 2x$$

$$= \frac{2x}{3\left(\sqrt[3]{x^2-5}\right)^2}, \quad x \neq \pm\sqrt{5}$$

b) $g(x) = \sqrt{8x-1} = (8x-1)^{\frac{1}{2}}$

$$g'(x) = \frac{1}{2}(8x-1)^{-\frac{1}{2}}\cdot 8 = 4\cdot\frac{1}{(8x-1)^{\frac{1}{2}}} = \frac{4}{\sqrt{8x-1}}, \quad x > \frac{1}{8}$$

16.10 $f(x) = \sqrt{x} - \sqrt{x+4} = x^{\frac{1}{2}} - (x+4)^{\frac{1}{2}}$ $\left| \begin{matrix} x \geq 0 \\ x+4 \geq 0 \Leftrightarrow x \geq -4 \end{matrix} \right\}$ $x \geq 0$

f on jatkuva $\overset{\text{kun } x \geq 0}{\text{ja}}$ derivoituva kun $x > 0$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}(x+4)^{-\frac{1}{2}}\cdot 1$$

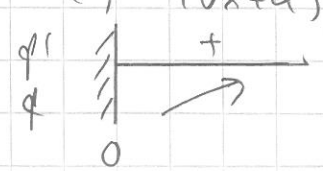
$$= \frac{1}{2}\cdot\frac{1}{x^{\frac{1}{2}}} - \frac{1}{2}\frac{1}{(x+4)^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x+4}} = 0 \quad | \cdot 2\sqrt{x}\sqrt{x+4}$$

$$\Leftrightarrow \sqrt{x+4} - \sqrt{x} = 0$$

$$\Leftrightarrow \sqrt{x+4} = \sqrt{x} \quad | ()^2$$

$$\Leftrightarrow (\sqrt{x+4})^2 = (\sqrt{x})^2 \Leftrightarrow x+4 = x \Leftrightarrow 0 = 4$$ ↯

$f'(1) = \frac{1}{2\sqrt{1}} - \frac{1}{2\sqrt{5}} \approx 0{,}28 > 0$



f on aidosti kasvava kun $x \geq 0$