b) $\lim_{h \to 0} \frac{f(1-h)-f(1)}{h} = \lim_{h \to 0} -\underbrace{\frac{f(1+(-h))-f(1)}{-h}}_{\to f'(1)} = -f'(1) = -2$

c) $\lim_{h \to 0} \frac{f(1+3h)-f(1)}{h} = \lim_{h \to 0} 3 \cdot \underbrace{\frac{f(1+3h)-f(1)}{3h}}_{\to f'(1)} = 3f'(1) = 3 \cdot 2 = 6$

d) $\lim_{h \to 0} \frac{f(1+h)-f(1-h)}{h} = \lim_{h \to 0} \frac{f(1+h)-f(1)+f(1)-f(1-h)}{h}$

$= \lim_{h \to 0} \left(\frac{f(1+h)-f(1)}{h} + \frac{f(1)-f(1-h)}{h} \right)$

$= \lim_{h \to 0} \left(\underbrace{\frac{f(1+h)-f(1)}{h}}_{\to f'(1)} + \underbrace{\frac{f(1-h)-f(1)}{-h}}_{\to f'(1)} \right) = 2f'(1) = 2 \cdot 2 = 4$

6. Polynomifunktion derivaatta

Esim. $f(x) = x^2$

$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a} \frac{x^2-a^2}{x-a} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x \to a} \frac{\cancel{(x-a)}(x+a)}{\cancel{x-a}}$

$= \lim_{x \to a} (x+a) = a + a = 2a$

$f'(2) = 2 \cdot 2 = 4$

$f'(5) = 2 \cdot 5 = 10$

$f'(12) = 2 \cdot 12 = 24$

$f'(\pi) = 2\pi$

$\vdots$

$f'(x) = 2x$ funktion f derivaattafunktio

Merk. $D\, x^2 = 2x$

Derivoimissääntöjä:

1. $D\, k = 0$, k vakio
2. $D\, x^n = n x^{n-1}$
3. $D(k \cdot f(x)) = k\, D f(x) = k f'(x)$, k vakio
4. $D(f(x) + g(x)) = D f(x) + D g(x) = f'(x) + g'(x)$