5.7 $f(x) = x^2 - x$

a) $k_k = \dfrac{\Delta y}{\Delta x} = \dfrac{f(3)-f(1)}{3-1} = \dfrac{(3^2-3)-(1^2-1)}{2} = \dfrac{6}{2} = \underline{3}$

b) $f'(2) = \lim\limits_{x\to 2} \dfrac{f(x)-f(2)}{x-2} = \lim\limits_{x\to 2} \dfrac{(x^2-x)-(2^2-2)}{x-2} = \lim\limits_{x\to 2} \dfrac{x^2-x-2}{x-2}$

$\overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 2} \dfrac{1\cdot(x-(-1))\cancel{(x-2)}}{\cancel{x-2}} = \lim\limits_{x\to 2} (x+1) = 2+1 = \underline{3}$

$\left[\; x^2-x-2=0 \iff x = \begin{cases} -1 \\ 2 \end{cases} \;\right]$

5.8 $f(x) = \dfrac{3}{x}$

$f'(-1) = \lim\limits_{x\to -1} \dfrac{f(x)-f(-1)}{x-(-1)} = \lim\limits_{x\to -1} \dfrac{\frac{3}{x} - \frac{3}{-1}}{x+1} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to -1} \dfrac{\frac{3}{x} + \frac{3x}{x}}{x+1}$

$= \lim\limits_{x\to -1} \dfrac{\frac{3+3x}{x}}{x+1} = \lim\limits_{x\to -1} \dfrac{3+3x}{x(x+1)}$

$\left[\; \dfrac{\frac{a}{b}}{c} = \dfrac{a}{b}\cdot\dfrac{1}{c} = \dfrac{a}{bc} \right.$

$= \lim\limits_{x\to -1} \dfrac{3\cancel{(1+x)}}{x\cancel{(x+1)}} = \lim\limits_{x\to -1} \dfrac{3}{x}$

$\dfrac{a}{\frac{b}{c}} = a\cdot\dfrac{c}{b} = \dfrac{ac}{b}$

$= \dfrac{3}{-1} = \underline{-3}$

5.21 g jatkuva kohdassa 0, $f(x) = x\,g(x)$ $\quad (f(0) = 0\cdot g(0) = 0)$

a) $f'(0) = \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0} \dfrac{x\,g(x)-0}{x} = \lim\limits_{x\to 0} \dfrac{x\,g(x)}{x} = \lim\limits_{x\to 0} g(x) = \underline{g(0)}$

g jatkuva kohdassa 0

b) $g(x) = |x|+1$

$\lim\limits_{x\to 0} g(x) = \lim\limits_{x\to 0}(|x|+1) = |0|+1 = 1 = g(0) \Rightarrow g$ jatkuva kohdassa 0

Siten $f'(0) = g(0) = \underline{1}$

5.22 a) $f(x) = x^2$

$f'(1) = \lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0} \dfrac{(1+h)^2-1^2}{h} = \lim\limits_{h\to 0} \dfrac{1^2+2\cdot 1\cdot h+h^2-1^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{2h+h^2}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{h}(2+h)}{\cancel{h}} = \lim\limits_{h\to 0} (2+h) = 2+0 = \underline{2}$