

i) tarketaan $f(a)$ jos mahdollista

ii) jos ei mahdollista (esim. tilanne $\frac{0}{0}$), muutetaan (supistetaan, laajennetaan, ...) $f(x)$ muotoa jolloin voidaan sijoittaa $x=a$

Esim. a) $\lim_{x \rightarrow -2} \frac{3x^2 + 12x + 12}{6x + 12} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \rightarrow -2} \frac{3(x^2 + 4x + 4)}{6(x+2)} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \rightarrow -2} \frac{3(x+2)^2}{2 \cancel{6}(x+2)}$

$$= \lim_{x \rightarrow -2} \frac{x+2}{2} = \frac{-2+2}{2} = \frac{0}{2} = \underline{0}$$

b) $\frac{x^2 + x - 6}{x^2 + 3x} \stackrel{\left(\frac{0}{0}\right)}{=} \frac{1 \cdot \cancel{(x-2)}(x-3)}{x \cancel{(x+3)}} = \frac{x-2}{x} \xrightarrow{x \rightarrow -3} \frac{-3-2}{-3} = \frac{-5}{-3} = \underline{\frac{5}{3}}$

$\Gamma x^2 + x - 6 = 0 \Leftrightarrow x = \begin{cases} 2 \\ -3 \end{cases}$

2.4 a) $\lim_{x \rightarrow 6} \frac{x^2 - 6x}{6 - x} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \rightarrow 6} \frac{x(x-6)}{6-x} = \lim_{x \rightarrow 6} \frac{\cancel{x(x-6)}}{-(x-6)} = \lim_{x \rightarrow 6} (-x) = \underline{-6}$

b) $\frac{x^2 - 16}{x - 4} \stackrel{\left(\frac{0}{0}\right)}{=} \frac{x^2 - 4^2}{x - 4} = \frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}} = x+4 \xrightarrow{x \rightarrow 4} 4+4 = \underline{8}$

2.6 a) $\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x^2 - 1} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \rightarrow -1} \frac{(x+1)^2}{(x-1)(x+1)} = \lim_{x \rightarrow -1} \frac{x+1}{x-1} = \frac{-1+1}{-1-1} = \frac{0}{-2} = \underline{0}$

b) $\lim_{x \rightarrow 1} \frac{x^4 - 1}{2 - 2x} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \rightarrow 1} \frac{(x^2)^2 - 1^2}{2(1-x)} = \lim_{x \rightarrow 1} \frac{\cancel{(x^2-1)}(x^2+1)}{2(1-x)} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x+1)(x^2+1)}{-2\cancel{(x-1)}}$

$$= \lim_{x \rightarrow 1} \frac{(x+1)(x^2+1)}{-2} = \frac{(1+1) \cdot (1^2+1)}{-2} = \frac{4}{-2} = \underline{-2}$$

2.20 a) $\lim_{x \rightarrow -2} \frac{2x^2 + x + a}{x+2}$ on olemassa

Koska nimittäjä $x+2 \rightarrow 0$, myös osittaja $\rightarrow 0$ (rajalle on oltava tilanne $\frac{0}{0}$, jos olisi esim. $\frac{5}{0} = \pm\infty$ ja raja-arvoa ei olisi olemassa)

$$x = -2: 2 \cdot (-2)^2 - 2 + a = 0 \Leftrightarrow 8 - 2 + a = 0 \Leftrightarrow a = \underline{-6}$$

Tällöin $\frac{2x^2 + x - 6}{x+2} \stackrel{\left(\frac{0}{0}\right)}{=} \frac{2 \cancel{(x-2)}(x-\frac{3}{2})}{\cancel{x+2}} = 2(x-\frac{3}{2}) = 2x-3$

$$\xrightarrow{x \rightarrow -2} 2 \cdot (-2) - 3 = \underline{-7}$$

$\Gamma 2x^2 + x - 6 = 0 \Leftrightarrow x = \begin{cases} -2 \\ \frac{3}{2} \end{cases}$