23.16 <u>Väite</u> $e^{2x} \geq 1+2x$, $x \in \mathbb{R}$

<u>Tod.</u> väite $\Leftrightarrow$ $\underbrace{e^{2x}-1-2x}_{=f(x)} \geq 0$

$= f(x)$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = e^{2x}\cdot 2 - 2 = 0$

$\Leftrightarrow e^{2x}\cdot 2 = 2 \quad |:2 \quad \Leftrightarrow e^{2x} = 1 \quad | \ln$

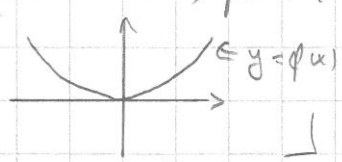
$\Leftrightarrow \ln e^{2x} = \ln 1 \quad \Leftrightarrow 2x = 0 \quad |:2 \quad \Leftrightarrow x = 0$

		0	
f'	$-$		$+$
f	$\searrow$		$\nearrow$
		min	

$f'(-1) \approx -1{,}73 < 0$, $f'(1) \approx 12{,}8 > 0$

pienin arvo: $f(0) = e^0 - 1 - 0 = 1 - 1 = 0$

$\Rightarrow f(x) \geq 0$ aina $\Rightarrow$ väite m.o.t.



23.17 $f(x) = (x^2 - x - 5)\cdot e^{-x}$

f jatk. ja deriv. kun $x \geq 0$

$f'(x) = (2x-1)e^{-x} + (x^2-x-5)\cdot e^{-x}\cdot(-1) = 0$

$\Leftrightarrow e^{-x}\left[(2x-1) - (x^2-x-5)\right] = 0$

$\Leftrightarrow e^{-x}(-x^2+3x+4) = 0$

$\Leftrightarrow \underbrace{e^{-x}}_{>0 \text{ aina}} = 0$ ei tai $-x^2+3x+4 = 0 \quad \Leftrightarrow x = \begin{cases} -1 \\ 4 \end{cases}$

	0		4	
f'		$+$		$-$
f		$\nearrow$		$\searrow$
			max	

$f'(1) \approx 2{,}7 > 0$

$f'(5) \approx -0{,}04 < 0$

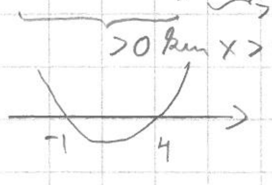
suurin arvo: $f(4) = (4^2-4-5)e^{-4} = \underline{7e^{-4}}$

$\left(= \dfrac{7}{e^4}\right)$

$f(0) = -5e^0 = -5$

$f(x) = \underbrace{(x^2-x-5)}_{>0 \text{ kun } x>4}\underbrace{e^{-x}}_{>0} > 0$ aina kun $x > 4$

$\Rightarrow$ <u>pienin arvo:</u> $\underline{f(0) = -5}$



23.23 <u>Väite</u> $x^x - e^{x-1} \geq 0$, $x \geq 1$

<u>Tod.</u>

$x^x - e^{x-1} \geq 0$

$\Leftrightarrow e^{\ln x^x} - e^{x-1} \geq 0$

$\Leftrightarrow e^{x\ln x} - e^{x-1} \geq 0$

$\Leftrightarrow e^{x\ln x} \geq e^{x-1} \quad | \ln$ (*)

$Dx^3 = 3x^2$

$D3^x = 3^x \ln 3$

$Dx^x = ?$

(*)