

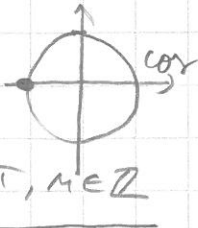
19,21

$$f(x) = \frac{5}{4 + 3 \underbrace{\cos 2x}_{-1 \leq \cos 2x \leq 1}}$$

minimitejään pienu arvo: $4 + 3 \cdot (-1) = 1$ \Rightarrow nimittäjä $\neq 0$ arvo $\Rightarrow x \in \mathbb{R}$

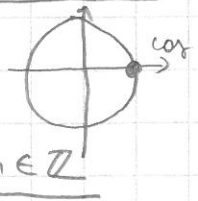
suurin arvo: $\frac{5}{4 + 3 \cdot (-1)} = \frac{5}{1} = 5$ kun $\cos 2x = -1$

$$\Leftrightarrow 2x = \pi + M2\pi \quad | :2 \quad \Leftrightarrow x = \frac{\pi}{2} + M\pi, M \in \mathbb{Z}$$



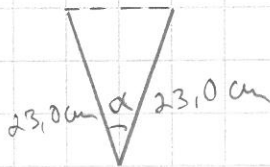
pienu arvo: $\frac{5}{4 + 3 \cdot 1} = \frac{5}{7}$ kun $\cos 2x = 1$

$$\Leftrightarrow 2x = M2\pi \quad | :2 \quad \Leftrightarrow x = M\pi, M \in \mathbb{Z}$$



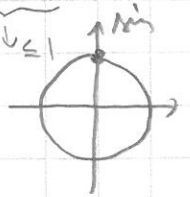
20. Suvelustehetäie siini- ja kosinifunktioista

20,4



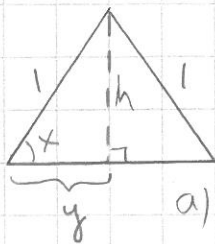
puite-ala: $A(x) = \frac{1}{2} \cdot 23,0 \text{ cm} \cdot 23,0 \text{ cm} \cdot \underbrace{\sin \alpha}_{0 \leq \sin \alpha \leq 1}$

$A(x)$ on suurin kun $\sin \alpha = 1 \Rightarrow \alpha = 90^\circ$



jälön tilavuus: $V = A \cdot h = \frac{1}{2} \cdot 2,30 \text{ dm} \cdot 2,30 \text{ dm} \cdot 30 \text{ dm}$
 $= 79,35 \text{ dm}^3 = \underline{79 \text{ l}}$

20,8



$$\begin{cases} \sin x = \frac{h}{1} = h \\ \cos x = \frac{y}{1} = y \end{cases}$$

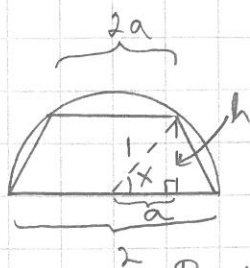
a) Puite-ala: $A(x) = \frac{1}{2} \cdot 2y \cdot h = yh = \underline{\cos x \cdot \sin x}$

b) $A(x) = \cos x \cdot \sin x = \frac{1}{2} \cdot \frac{2 \sin x \cos x}{\sin 2x} = \frac{1}{2} \underbrace{\sin 2x}_{0 \leq \sin 2x \leq 1}$

suurin arvo: $\frac{1}{2} \cdot 1 = \frac{1}{2}$ kun $2x = 90^\circ \quad | :2 \quad \Leftrightarrow x = 45^\circ$

Jälön kanta: $2y = 2 \cos x = 2 \cdot \cos 45^\circ = 2 \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$
 $= \frac{2\sqrt{2}}{2} = \underline{\sqrt{2}}$

20,9



$$\begin{cases} \cos x = \frac{a}{1} = a \\ \sin x = \frac{h}{1} = h \end{cases}$$

Pudimunnin muotoisen puite-ala:

$$A(x) = \frac{2a+2}{2} \cdot h = (a+1)h = (\cos x + 1) \sin x, \text{ Ajäls. ja deriiv. väl. } [0, \frac{\pi}{2}]$$